

RESTRICTED

INFORMATION REPORT

COUNTRY USSR
SUBJECT Scientific Publication
PLACE ACQUIRED
DATE ACQUIRED See below.

DATE DISC 12 March 1946
NO. OF PAGES
NO. OF ENCL 25X1A
SUPPLEMENT TO REPORT NO.

GRADING OF SOURCE						COLLECTOR'S PRELIMINARY GRADING OF CONTENT					
COMPLETELY RELIABLE	USUALLY RELIABLE	FAIRLY RELIABLE	NOT USUALLY RELIABLE	NOT RELIABLE	UNKNOWN	PROBABLY TRUE	POSSIBLY TRUE	DOUBTFUL	PROBABLY FALSE	POSSIBLY FALSE	UNKNOWN
1	2	3	4	5	6	1	2	3	4	5	6

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANS OF THE DISPOSABLE ART OF THE UNITED STATES. IT IS UNCLASSIFIED AND ITS DISSEMINATION IS UNRESTRICTED. IT IS NOT TO BE RELEASED TO ANY PERSON OR ENTITY WITHOUT THE AUTHORITY OF THE DISSEMINATION AUTHORITY. REPRODUCTION OF THIS FORM IS PROHIBITED. INFORMATION CONTAINED HEREIN IS UNCLASSIFIED. DATE 10/1/98 BY 6032

THIS IS UNEVALUATED INFORMATION FOR THE RESEARCH USE OF TRAINED INTELLIGENCE PERSONNEL

SOURCE DOCUMENTARY.

On file with CIA are two copies of Isoplas London (Doklady), Volume No 6, dated 1946, published by the Academy of Sciences of the USSR.

Contents

Mathematiques	
GI Egdin. On an Effective Method of Calculation of the Mathematical Expectations of Central Sample Moments.	447
V Nienytrik. Les systemes dynamiques generaux	451
A Norden. On the Invariants of Conjugated Nets.	455
NA Shanin. On Intersection of Open Subsets in the Product of Topological Spaces.	459
Theorie De L'Elasticite	
VV Novozhilov. A Generalization of the Method of Complex Displacements to the Non-homogeneous Problem of the Theory of Shells	503
Physique	
MI Eshuev. Effect of the Hydrogen Bond on the Frequencies of the Carbonyl Grouping in the Fatty Acids.	507
BM Hochberg and BE Sandberg. Ionization of Gases and their Breakdown Strength	511
Physique De Globe	
AB Stockmann. A Tentative Indirect Determination of the Velocity of the Trade Winds in the Equatorial Part of the Pacific	515
Physique Appliquees	
XP Stanivkovich. Unidimensional Scattering of Detonation Products in the Case of Local Action Explosives.	519
Electroite	
IS Bruk, Corresponding Member of the Academy. A Device for the Solution of Ordinary Differential Equations.	523

CLASSIFICATION				RESTRICTED			
STATE	X NAVY	E USSR		DIST. 1			
ARMY	E AIR	E					

REFERENCE CENTER LIBRARY

RESTRICTED

RESTRICTED

25X1A

Chimie

A A Petrov. Condensation of Diene Hydrocarbons with Butynes 527

Chimie Du Globe

R V Teis. Variation in Deuterium Concentration in the Process of
Melting of Ice 529

Chimie Biologique

M A Rosanova. Daily Variation in the Accumulation of Ascorbic Acid by
Leaves and Fruits of Active and Inactive Species of Wild Roses 533
A G Toashchevikova. Proteinase in Plants as Affected by Biogenic
Stimulants 537

Chimie Agricole

M I Kalinkevich. On the Role of Phosphoric Acid in the Accumulation of
Essential Oils in the Leaves of Ocimum canum 541

Geologie

V V Butsura. The post-Pleistocene Age of the East Carpathians 545

Mineralogie

D S Beliankin, Member of the Academy, V V Lapin and J P Simanov. On
Calcareous Hexaaluminate from Zestafoni (Georgia). 549

Petrographie

N A Ignatiev and A M Kuznetsov. On the Composition and Properties of
"Vaps" 553

Paleontologie

S Rosovskaya. On the Genus Hamifusulina Moeller. 557

Genetique

E N Volotov. The Effect of Polyploidy upon the Tapetum in Poppy. 561

Ecologie

A A Malyshev. Crop Plants under Extreme Conditions of High Altitudes
of the North Caucasus and the North-East Altai 565

Cytologie

G V Kharlova. Observations on the Nucleus at Interkinesis. 569

Microbiologie

M N Meissel and N P Trofimova. On the Utilization of Volatile Biocataly-
tic Substances by Microorganisms 573

Morphologie Experimentale

T Dettlaff. On the Question as to the Existence of the "Nerve" Layer of
the Ectoderm 577

-end-

RESTRICTED

RESTRICTED

25X1A

Approved For Release 2002/07/29 : CIA-RDP80-00926A000200010072-1

25X1A

COMPTES RENDUS (DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

25X1A

1946

VOLUME LIII

№ 6

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

Approved For Release 2002/07/29 : CIA-RDP80-00926A000200010072-1

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours. Neuf numéros composent un volume pourvu d'une table des matières. Il y a quatre volumes par année.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» en de courtes communications précisent l'essentiel des travaux en préparation. Ces articles, ne dépassant pas en règle générale quatre pages, sont donnés en anglais ou en français. Ils embrassent les sciences mathématiques, physiques, naturelles et appliquées, mettant au jour les résultats de recherches scientifiques en cours ou qui viennent d'être achevées, et représentant ainsi les plus récentes données de l'investigation scientifique.

L'auteur a droit à 100 exemplaires de son article.

Adressez les manuscrits à la Rédaction des «Comptes Rendus», Moscou, Wolkhonka, 14.

30 Août

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

D. Bellankin, de l'Académie, V. Chlopin, de l'Académie, A. Frumkin, de l'Académie, A. Kolmogoroff, de l'Académie (vice rédacteur), L. Orbell, de l'Académie (vice rédacteur), A. Richter, de l'Académie, V. Rodionov, de l'Académie, S. Soboleff, de l'Académie, S. Vavilov, de l'Académie (rédacteur en chef)

NOUVELLE SÉRIE

14-me année

Paraissant tous les dix jours

1946

VOLUME LIII, N° 6

TABLE DES MATIÈRES

	Pages
MATHÉMATIQUES	
G. I. Egudin. On an Effective Method of Calculation of the Mathematical Expectations of Central Sample Moments	487
V. Niemytzki. Les systèmes dynamiques généraux	491
A. Norden. On the Invariants of Conjugated Nets	495
N. A. Shanin. On Intersection of Open Subsets in the Product of Topological Spaces	499
THÉORIE DE L'ÉLASTICITÉ	
V. V. Novozhilov. A Generalization of the Method of Complex Displacements to the Non-homogeneous Problem of the Theory of Shells	503
PHYSIQUE	
M. I. Batuev. Effect of the Hydrogen Bond on the Frequencies of the Carbonyl Grouping in the Fatty Acids	507
B. M. Hochberg and E. J. Sandberg. Ionization of Gases and their Breakdown Strength	511
PHYSIQUE DU GLOBE	
W. B. Stockmann. A Tentative Indirect Determination of the Velocities of the Trade Winds in the Equatorial Part of the Pacific	515

PHYSIQUE APPLIQUÉE

BLE

ILLEGIB

CHIMIE

A. A. Petrov. Condensation of Diene Hydrocarbons with Butynone . . . 527

CHIMIE DU GLOBE

R. V. Tels. Variation in Deuterium Concentration in the Process of Melting of Ice . . . 529

CHIMIE BIOLOGIQUE

M. A. Rosanova. Daily Variation in the Accumulation of Ascorbic Acid by Leaves and Fruits of Active and Inactive Species of Wild Roses . . . 533

A. G. Toshchevikova. Proteinase in Plants as Affected by Biogenic Stimulants . . . 537

CHIMIE AGRICOLE

M. I. Kalinkevich. On the Rôle of Phosphoric Acid in the Accumulation of Essential Oils in the Leaves of *Ocimum canum* . . . 541

GÉOLOGIE

V. V. Butzura. The post-Pleistocene Age of the East Carpathians . . . 545

MINÉRALOGIE

D. S. Bellankin, Member of the Academy, V. V. Lapin and J. P. Simanov. On Calcareous Hexaaluminate from Zestafoni (Georgia) . . . 549

PÉTROGRAPHIE

N. A. Ignatiev and A. M. Kuznetsov. On the Composition and Properties of «Vapso» . . . 553

PALEONTOLOGIE

S. Rosovskaya. On the Genus *Hemifusulina* Moeller . . . 557

GÉNÉTIQUE

E. N. Volotov. The Effect of Polyploidy upon the Tapetum in Poppy . . . 561

ÉCOLOGIE

A. A. Malyshev. Crop Plants under Extreme Conditions of High Altitudes of the North Caucasus and the North-East Altai . . . 565

CYTOLOGIE

G. V. Kharlova. Observations on the Nucleus at Interkinesis . . . 569

MICROBIOLOGIE

M. N. Melssel and N. P. Trofimova. On the Utilization of Volatile Biocatalytic Substances by Microorganisms . . . 573

MORPHOLOGIE EXPÉRIMENTALE

T. Dettlaff. On the Question as to the Existence of the «Nerve» Layer of the Ectoderm . . . 577

Traductions rédigées par D. Rakhmanov et T. Rogalina

Édition de l'Académie des Sciences de l'URSS
Moscou 1946

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

MATHEMATICS

ON AN EFFECTIVE METHOD OF CALCULATION
OF THE MATHEMATICAL EXPECTATIONS OF CENTRAL
SAMPLE MOMENTS

By G. I. EGUDIN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 11. IV. 1946)

We give a uniform operational method of successive calculation, for various r , of the mathematical expectations of central sample moments μ_r ($r=1, 2, \dots$), defined for repeated random samples of finite size n from an arbitrary universe. Such a mathematical expectation can be expressed directly through the parameters of the universe from which the samples are drawn. The results obtained enable us, moreover, to establish directly some properties of the central sample moments under more general conditions than those set up till now.

Let $F(x)$ be the integral probability law of a random variable in the universe. Suppose the integral $\int_{-\infty}^{\infty} x^r dF(x)$ exist for all desired positive integral values of r , and let m_r and \dot{m}_r be the initial moments of the universe and the initial moments of the sample⁽¹⁾, respectively. It is easy to see that the central moment of the universe

$$\mu_r = \int_{-\infty}^{\infty} (x - m_1)^r dF(x)$$

can be represented in the form

$$\mu_r = \sum_{h=0}^r C_r^h m_{r-h} m_1^h (-1)^h \quad (1)$$

The central sample moment, consequently, is

$$\dot{\mu}_r = \sum_{h=0}^r C_r^h \dot{m}_{r-h} \dot{m}_1^h (-1)^h \quad (2)$$

For repeated random samples we easily obtain with the use of formula (12) of our note⁽¹⁾

$$E(\dot{m}_k \dot{m}_1^q) = \frac{1}{n^q} O^q(m_0^{n-1} m_k) \quad (3)$$

where the differential operator O satisfies, besides usual rules of differentiation, the condition $O(m_h^a) = a m_h^{a-1} m_{h+1}$; we must put $m_0 = 1$ after the operator is applied.

Consequently,

$$E(\dot{\mu}_r) = \sum_{h=0}^r C_r^h \frac{1}{n^h} O^h(m_0^{n-1} m_{r-h}) (-1)^h \quad (4)$$

Leibnitz's formula, changing the order of summation and applying the equality

$$\sum_{h=1}^r C_h^i C_r^h n^{r-h} (-1)^h = C_r^i (n-1)^{r-i} (-1)^i \quad (5)$$

we put (4) in the form

$$E(\dot{\mu}_r) = \frac{1}{n^r} \sum_{h=0}^r C_r^h (n-1)^{r-h} O^{r-h}(m_0) O^h(m_0^{n-1}) (-1)^h \quad (6)$$

It is now almost evident that this can be represented in the form

$$E(\dot{\mu}_r) = \frac{1}{n^r} \Omega^r(m_0 M_0^{n-1}) \quad (7)$$

where Ω is a differential operator satisfying the usual rules of differentiating sums and products; as to m and M they behave differently with respect to Ω :

$$\Omega^r(U) = \Omega[\Omega^{r-1}(U)], \quad \Omega(m_k) = (n-1)m_{k+1}, \quad \Omega(M_k^n) = -\alpha M_k^{n-1} M_{k+1}$$

After applying the operator Ω it is necessary to put $m_0 = 1$, $M_k = m_k$. The expression of $E(\dot{\mu}_r)$ through m can be obtained easily (in comparison with all the methods used) by repeated application of formula (7). By putting $m_1 = 0$ and $m_k = \mu_k$ (for $k > 1$) in the formulae so obtained, we get the expression of $E(\dot{\mu}_r)$ through μ .

Formulae expressing $E(\dot{\mu}_r)$ through the parameters of the universe μ and m for some concrete values of r have been given more than once by a number of authors, and the correct results coincide with those obtained by using formula (7)*.

Lack of uniform algorithms, the cumbersome nature of the numerical methods employed, and the fact that the computations greatly increase in amount and become involved with increase of r , have led to an unusually large number of errors even in the works of skillful mathematicians.

By the cumulant of the universe we shall mean the quantities introduced first by Thiele(†)**

$$\lambda_r = \frac{1}{i^r} \left[\frac{d^r \ln \varphi(t)}{dt^r} \right]_{t=0}$$

where $\varphi(t) = \int_{-\infty}^{\infty} e^{itx} dF(x)$ is the characteristic function of the variable.

We see that $m_r = \varphi^{(r)}(0)$, that is, the initial moment m_r is expressed through the cumulant by the operational formula $m_r = O^r(U)$, where the differential operator O satisfies the additional conditions

$$O(U) = U\lambda_1, \quad O(\lambda_k^n) = \alpha \lambda_k^{n-1} \lambda_{k+1},$$

and after the operator is applied, it is necessary to put $U=1$. By an argument analogous to that expounded above we can establish the operational formula for successive expression of the mathematical expecta-

* See the list of the results, for instance, in (4).

** Thiele calls them semi-invariants. Most of the authors use the same term, but as follows from our note (1), the term is unsatisfactory: being invariant under translations of the origin, the cumulants, together with central moments and φ parameters introduced in (1), are particular cases of a wide class of semi-invariant quantities. The term "cumulant" is due to Fisher (3), who employs it along with the old term.

tions of central sample moments of various orders (for repeated samples) through the cumulants of the universe

$$E(\mu_r) = \frac{1}{n^r} \Omega(w) \quad (8)$$

The special conditions imposed here on the differential operator Ω are as follows

$$\begin{aligned} \Omega(w) &= (n-1)(\lambda_1 - \Delta_1)w \\ \Omega(\lambda_k^a) &= \alpha(n-1)\lambda_k^{a-1}\lambda_{k+1}, \quad \Omega(\Delta_k^a) = -\alpha\Delta_k^{a-1}\Delta_{k+1} \end{aligned}$$

After the operator is applied it is necessary to put

$$w = 1, \quad \Delta_k = \lambda_k$$

Using the above results it is easy to deduce a number of asymptotic formulae for the mathematical expectations of central sample moments defined for repeated samples of size n as n tends to infinity. Since $O^{r-h}(m_0) = m_{r-h}$ and $O^h(m_0^{h-1}) = (n-1)(n-2)\dots(n-h)m_1^h + \text{terms of lower degree in } n$, (6) and (4) imply

$$\begin{aligned} \lim_{n \rightarrow \infty} E(\mu_r) &= \lim_{n \rightarrow \infty} \sum_{h=0}^r C_r^h \frac{(n-1)^{r-h} (n-1)(n-2)\dots(n-h)}{n^r} \times \\ &\times m_{r-h} m_1^h (-1)^h = \sum_{h=0}^r C_r^h m_{r-h} m_1^h (-1)^h = \mu_r \end{aligned} \quad (9)$$

From (4) follows

$$\mu_r^2 = \sum_{h=0}^r C_r^h \sum_{i=0}^r C_r^i m_{r-h} m_{r-i} m_1^{h+i} (-1)^{h+i} \quad (10)$$

In the same way as (3) one can obtain

$$E(\mu_k \mu_i \mu_1^q) = \frac{1}{n^{q+1}} O^q [(n-1) m_0^{n-2} m_i m_k + m_0^{n-1} m_{k+i}] \quad (14)$$

Consequently,

$$\begin{aligned} E(\mu_r^2) &= \sum_{h=0}^r C_r^h \sum_{i=0}^r C_r^i \frac{1}{n^{h+i+1}} O^{h+i} [(n-1) m_0^{n-2} m_{r-h} m_{r-i} + \\ &+ m_0^{n-1} m_{2r-h-i}] (-1)^{h+i} \end{aligned} \quad (12)$$

For $\lim_{n \rightarrow \infty} E(\mu_r^2)$ only the terms containing the highest degrees of n are essential. They evidently form but a part of summands of the form $(n-1) O^{h+i}(m_0^{n-2}) m_{r-h} m_{r-i} = (n-1)(n-2)\dots(n-2-h-i) m_1^{h+i} m_{r-h} m_{r-i} + \text{terms of lower degree in } n$.

Hence passing to the limit in (12), we obtain

$$\lim_{n \rightarrow \infty} E(\mu_r^2) = \sum_{h=0}^r C_r^h \sum_{i=0}^r C_r^i m_1^{h+i} m_{r-h} m_{r-i} = \mu_r^2 \quad (13)$$

In conclusion two remarks are necessary:

1) Formula (12) can be used immediately for calculation of the mathematical expectations of the squares of central sample moments of various orders, provided the size of the repeated samples is finite. The calculations are not too difficult when r is not very large. Together with formula (7), this enables us to find $E(\mu_r^2) - [E(\mu_r)]^2$, the dispersion of the central sample moments, which is rather important for the statistics both from the practical and theoretical viewpoint. When n is sufficiently large and we can restrict ourselves to the terms of the order

n^{-1}, n^{-2}, \dots , we can determine the corresponding highest terms (with respect to n) in (12) by a method similar to that used for (13).

2) Unlike other proofs, such as that based on the so-called Mises's law of large numbers, our direct deduction (for any r) of formulae (9) and (13), which characterize the stability of central sample moments, does not need the existence of the probabilities $p_i = P(X = x_i)$ of every possible value of the random variable. We require only the existence

of $\int_{-\infty}^{\infty} x^k dF(x)$ for all the desired values of k , which takes place when the conditions imposed on F are far more general than those necessary for the existence of the probabilities p_i .

Received
11. IV. 1946.

REFERENCES

- ¹ G. E g u d i n, C. R. Acad. Sci. URSS, 48, No. 9 (1945). ² T. N. Thiele, Theory of Observations, 1903; Ann. Math. Statistics, 2, 2, 165 (1931). ³ R. Fisher, Proc. London Math. Soc., 2, 30, 199 (1929). ⁴ C. Y o r d a n, Statistique mathématique, Paris. ⁵ P. D w y e r, Ann. Math. Statistics, 8, 1, 21 (1937). ⁶ P. D w y e r, ibid., 9, 1, 1 (1938); 9, 2, 97 (1938).

MATHÉMATIQUES

LES SYSTÈMES DYNAMIQUES GÉNÉRAUX

Par V. NIEMYTZKI

(Présenté par A. N. Kolmogoroff, de l'Académie, le 21. III. 1946)

1. Dans la dernière dizaine d'années les savants américains Zippin et Montgomery ⁽¹⁾ ont étudié la théorie des groupes des transformations topologiques des espaces topologiques en eux-mêmes. D'ailleurs, ils ont presque toujours considéré le cas où le groupe G donné est un groupe topologique compact, et l'espace donné M est une multiplicité à n dimensions.

Dans une Note récente ⁽²⁾ Barbachine a indiqué la possibilité d'appliquer cette théorie à la généralisation de la notion de système dynamique abstrait de Birkhoff. D'ailleurs, il est certainement nécessaire de rejeter l'hypothèse que le groupe est compact et de la remplacer par une condition moins restrictive. Barbachine au lieu d'imposer des restrictions topologiques prend pour axiome que le groupe doit être partiellement ordonné. Dans ses hypothèses on peut obtenir des analogies profondes avec la théorie de Birkhoff.

La présente Note a pour but de montrer qu'en imposant au groupe une restriction topologique naturelle, à savoir en exigeant qu'il soit localement compact, nous pouvons non seulement transporter dans ce domaine général les théorèmes indiqués de Barbachine, mais obtenir encore une série de résultats purement topologiques qui caractérisent la disposition des «orbites» dans l'espace M .

J'ai indiqué la possibilité de construire une théorie de cette nature dans une conférence qui a eu lieu au cours des «conférences dédiées à Lomonosov» en 1945 et au cours d'un séminaire sous ma direction qui a eu lieu en automne de 1945.

2. Introduisons une définition. Soit G un groupe topologique et M un espace topologique. En suivant Zippin ⁽³⁾ nous dirons que G est un groupe effectif de transformations M , si les conditions suivantes sont remplies:

1°. Chaque élément $g \in G$ est un homéomorphisme $g(x)$ de l'espace M en lui-même.

2°. Si l'on considère $g(x)$ comme une fonction de g et de x , elle est continué par rapport à l'ensemble des variables.

3°. Si $g_1 \in G$ et $g_2 \in G$, alors pour tous les x on a $g_1[g_2(x)] = (g_1 g_2)(x)$.

4°. L'unité du groupe est le seul élément du groupe G tel que $e(x) = x$ pour tous les $x \in M$.

Définition. Un groupe localement compact G de transformations d'un espace métrique complet M sera nommé système dynamique général. Soit $x \in M$, alors l'ensemble des points $E_x = \{y = g(x)\}$, où g est un élément arbitraire du groupe G , sera dit «orbite» du point x , ou bien «ensemble orbital».

Comme G est localement compact, on peut trouver une suite dénombrable d'ensembles compacts $W_1, W_2, W_3, \dots, W_n, \dots$ jouissant des propriétés suivantes:

1°. W_i contient e^* pour chaque valeur de i .

2°. $W_{i+1} \supset W_i$; $W_{i+1} - W_i$ n'est pas vide.

3°. $\sum_{i=1}^{\infty} W_i = G$.

4°. Chaque sous-ensemble compact de G ne possède des points communs qu'avec un nombre fini d'ensembles W_i . La suite ainsi construite sera nommée suite épuisante.

Pour rapprocher notre théorie avec les résultats de Barbachine (*) remarquons que le groupe G peut être partiellement ordonné d'une manière très naturelle.

Nous dirons qu'un élément g_k suit un élément g_i si $g_k \subset W_i$, $g_i \in W_k$, où $i > k$.

Définition. Un ensemble $\Omega_x \subset R$ sera dit ensemble de points limites dynamiques pour une orbite E_x , si pour chaque $y \in \Omega_x$ il existe une suite divergente (n'ayant aucun point limite) d'éléments $g_1, g_2, \dots, g_n, \dots$ du groupe G telle que $\lim_{n \rightarrow \infty} g_n(x) = y$.

Chaque série ω ou z de Barbachine (*) est une suite divergente pour notre mode d'ordonner partiellement. Inversement, si $g_1, g_2, \dots, g_n, \dots$ est une suite divergente, on peut extraire de chacune de ces suites partielles une série ω . Cela étant, on peut dire que le théorème 1 est une conséquence du théorème 1 de Barbachine**.

Théorème 1. L'ensemble Ω_x est formé d'«orbites» des points qui le composent.

Ensuite les théorèmes suivants ont lieu.

Théorème 2. Les ensembles orbitaux sont situés dans l'espace R d'une manière continue, ce qui veut dire que

1°. Ils n'ont pas de points communs.

2°. Si $p \in \bar{E}_q$, où \bar{E}_q est la fermeture de l'ensemble orbital du point q , on a $E_p \subset E_q$.

3°. Si la suite $p_1, p_2, \dots, p_n, \dots$ tend vers p , on a $E_p \subset \text{lt } E_{p_n}$, où $\text{lt } E_{p_n}$ est la limite topologique inférieure des ensembles orbitaux $\{\bar{E}_{p_n}\}$ (*).

Pour prolonger la construction de la théorie il faut démontrer le lemme suivant:

Lemme. Si l'ensemble orbital E_p du point p n'est pas un ensemble fermé, on a

$$\Omega_p = E_p - E_p$$

La démonstration du lemme est presque la même que dans la théorie ordinaire des systèmes dynamiques.

Définition. Un point $p \in M$ est dit stable au sens de Poisson si $\Omega_p \cdot E_p \neq 0$ et $\bar{E}_p - E_p \neq 0$.

Théorème 3. Si p est stable au sens de Poisson, dans chaque voisinage d'un point $y \in E_p$ il existe des points qui n'appartiennent pas à E_p .

Démonstration. Tout d'abord il suit du théorème 2 que l'on a $y \in E_p \subset \Omega_p$. Soit $W_1, W_2, \dots, W_n, \dots$ une suite épuisante. Considérons les ensembles $K_1, K_2, \dots, K_n, \dots$ où $K_n = f(p, W_n)$ est l'ensemble des images du point p dans les transformations qui forment W_n . Comme $\sum_{n=1}^{\infty} W_n = G$, on a $\sum_{n=1}^{\infty} K_n = E_p$, et d'ailleurs les K_n sont des ensembles compacts de l'espace M . Soit maintenant $\varepsilon > 0$ un nombre positif aussi petit que l'on veut.

* On désigne par e l'unité dans G .

** Tous les théorèmes énoncés ici peuvent être démontrés sans faire l'hypothèse que le groupe est partiellement ordonné et sans recourir à la Not. de Barbachine.

Prenons d'abord dans le groupe G un élément g_{n_1} tel que

1°. $g_{n_1} \in G - W_1$, alors $p_n = g_{n-1}(p)$ n'appartient pas à K_1 ;

2°. $\rho(p_n, y) < \varepsilon$.

Prenons maintenant ε_1

$$\varepsilon_1 < \min \left\{ \frac{\varepsilon}{2}; \varepsilon - \rho(y, p_{n_1}); \frac{1}{2} \rho(p_{n_1}, K) \right\}$$

Supposons que l'on a déjà choisi les éléments $g_{n_{s-1}} \in G$, $p_{n_{s-1}} \in M$ et le nombre ε_{s-1} . Prenons maintenant un g_{n_s} de telle manière que l'on ait

1°. $g_{n_s} \in G - W_{n_{s-1}}$.

2°. $p_{n_s} = g_{n_s}(p) \in S(p_{n_{s-1}}, \varepsilon_{s-1})$.

Un tel choix est possible puisque $p_{n_{s-1}} \in \Omega_p$.

En vertu du choix de g_{n_s} on a $\rho(p_{n_s}, K_{n_{s-1}}) \neq 0$.

Posons maintenant

$$\varepsilon_s \leq \min \left\{ \frac{\varepsilon_{s-1}}{2}; \varepsilon_{s-1} - \rho(p_{n_{s-1}}, p_{n_s}); \frac{1}{2} \rho(p_{n_s}, K_{n_s}) \right\}.$$

Prolongeons ce procédé indéfiniment.

Nous obtenons ainsi une suite $p_{n_1}, p_{n_2}, \dots, p_{n_s}, \dots$ vérifiant le critère de Cauchy, donc M étant complet, cette suite est convergente.

Si $p_0 = \lim p_{n_s}$, on a $p_0 \in \Omega_p$, $\rho(p_0, y) < \varepsilon$.

Ce point n'appartient pas à E_p , ainsi qu'il résulte de sa construction.

Il est facile de voir que les points stables au sens de Poisson sont aussi stables au sens de Barbachine (*) quand on admet notre mode d'ordonner partiellement; donc, si M est compact, le théorème intéressant de Barbachine sur le centre subsiste pour les systèmes dynamiques généraux. Les autres théorèmes sur la famille des trajectoires stables au sens de Poisson se démontrent comme dans la théorie ordinaire des systèmes dynamiques.

3. Définition. Un point p est nommé récurrent si pour chaque $\varepsilon > 0$ on peut trouver un ensemble compact \tilde{W} d'éléments du groupe G contenant l'unité du groupe et tel que pour chaque point $q \in E_p$ l'ensemble $f(q, \tilde{W})$, c'est-à-dire l'ensemble des images du point q , dans toutes les transformations formant \tilde{W} , approche tout l'ensemble E_p à ε près, c'est-à-dire, pour chaque point $r \in E_p$ il existe un $g \in \tilde{W}$ tel que $\rho(g(q), r) < \varepsilon$.

On aurait pu montrer que la propriété de récurrence indiquée dans cette définition est une propriété caractéristique pour les points récurrents au sens de Barbachine, donc dans le cas où M est compact les théorèmes de Barbachine s'appliquent aux systèmes dynamiques généraux. Cependant nous donnerons, en qualité d'exemple, une démonstration indépendante* de l'un de ces théorèmes.

Théorème 4. Si \mathfrak{M} est formé d'orbites entières, s'il est compact, fermé et l'orbite de chacun de ses points est partout dense, alors tous ses points sont récurrents.

Soit $W_1, W_2, \dots, W_n, \dots$ une suite épuisante d'ensembles compacts, et soit, contrairement à l'hypothèse, $r \in \mathfrak{M}$ un point non récurrent. Il existe alors un $\varepsilon > 0$ tel que pour chaque ensemble compact $\tilde{W} \subset G$ contenant l'unité du groupe il existe un point $q \in E_r$ tel que $f(q, \tilde{W})$ n'approche pas E_r à ε près. Prenons en qualité d'ensembles compacts \tilde{W} les ensembles $W_1, W_2, \dots, W_k, \dots$

Nous aurons deux suites de points de l'orbite E_r :

$$q_1, q_2, \dots, q_n, \dots \quad \text{et} \quad r_1, r_2, \dots, r_n, \dots$$

* Les démonstrations de Barbachine ne sont pas encore publiées.

tels que $\varphi(r_n, f(q_n, W_n)) = \varepsilon$. Supposons que $r_n \rightarrow r$ et $q_n \rightarrow q$. Nous démontrons que $\varphi(r, E_q) \geq \frac{\varepsilon}{4}$, ce qui contredit à l'hypothèse que les E_q sont partout denses dans \mathfrak{M} .

On démontre le théorème inverse comme dans la théorie ordinaire de la stabilité des systèmes.

4. Comme exemple naturel de la théorie considérée on peut prendre la famille des solutions du système d'équations de Pfaff, pourvu que les conditions d'intégrabilité soient remplies.

Manuscrit reçu
le 21. III. 1946.

LITTÉRATURE CITÉE

¹ D. Montgomery and L. Zippin, Duke Math. Journ., **4**, 138, p. 364; Ann. of Math., **41**, 778 (1940); D. Montgomery, ibid., **46**, 383 (1945). ² E. Barbachine, C. R. Acad. Sci. URSS, **51**, No. 1 (1946). ³ L. Zippin, Lectures in Topology, p. 191, 1941. ⁴ F. Hausdorff, Mengenlehre, 2. Aufl., S. 146.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

MATHEMATICS

ON THE INVARIANTS OF CONJUGATED NETS

By A. NORDEN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 5. VII. 1946)

If a surface of a three-dimensional projective space with points

$$x^a = x^a(u^1, u^2) \quad (1)$$

is normalized, then at every of its points is defined a straight line x^a , X^a , and in the tangential plane ξ_a —the line of its intersection with the plane Ξ_a . Defining the first of them by the planes

$$\eta_{ai} = \partial_i \xi_a - \lambda_i \xi_a^* \quad (2)$$

containing it, and the second by the two points

$$y_i^a = \partial_i x^a - l_i x^a \quad (3)$$

and forming the equations

$$\partial_j y_i^a - g_{ij}^k y_k^a = l_j y_i^a + p_{ij} x^a + b_{ij} x^a \quad (4)$$

$$\partial_j \eta_{ai} - l_{ij}^k \eta_{ak} = \lambda_j \eta_{ai} + \pi_{ij} \eta_a + b_{ij} \Xi_a \quad (5)$$

we define the inner geometries of the normalized surface with symmetrical connexion coefficients g_{ij}^k and l_{ij}^k , respectively⁽¹⁾.

Every covariant vectors t_i and τ_i define straight lines, one of which lies in the plane ξ_a and passes through the points

$$z_i^a = y_i^a + t_i x^a \quad (6)$$

and the other, passing through the point x^a , lies in the intersection of the planes

$$\xi_{ai} = \eta_{ai} + \tau_i \xi_a \quad (7)$$

If the normalizing straight lines are changed, the inner geometries, the tensors p_{ij} , π_{ij} , as well as the tensors t_i and τ_i defining the straight lines invariantly connected with the surface, undergo certain transformations.

Suppose that a conjugated net is defined by the equation

$$a_{ij} du^i du^j = 0 \quad (8)$$

Denote by \tilde{a}^{ij} the tensor reciprocal to the tensor a_{ij} of the net; by ε^{ij} , the bivector determined by the condition $\varepsilon^{ik} \varepsilon^{jl} a_{kl} = \tilde{a}^{ij}$; by ε_{ij} , the bivector determined by the condition $\varepsilon^{ik} \varepsilon_{kj} = \delta_j^i$. Consider now the tensors

$$P_{ij} = \frac{1}{2} (p_{ij} + \nabla_j t_i - t_i t_j), \quad \Pi_{ij} = \frac{1}{2} (\pi_{ij} + \nabla_j \tau_i - \tau_i \tau_j) \quad (9)$$

* In the sequel we suppose that the Latin indices assume the values 1, 2 and the Greek ones, the values 1, 2, 3, 4.

assuming that the covariant differentiation in their expression is carried out with respect to the connexion of the first and second kind, respectively. In this case the quantities

$$\tilde{a}^{ij}P_{ij}, \quad \varepsilon_{ij}P_{ij}, \quad \tilde{a}^{ij}\Pi_{ij}, \quad \varepsilon_{ij}\Pi_{ij} \quad (10)$$

will be invariants of the change of normalization. We shall say that the weight of these invariants is equal to -1 , since they are multiplied by λ^{-1} when the tensor a_{ij} is replaced by the tensor λa_{ij} .

In order to obtain invariant expressions connected only with the choice of the conjugated net, we may assume that the straight lines determined by the vectors t_i and τ_i coincide with Green's ray and axis of this net ⁽²⁾. The latter of these assertions will hold if

$$t_k = \frac{1}{2} \tilde{a}^{ij} \left(\nabla_i a_{jk} - \frac{1}{2} \nabla_k a_{ij} \right) \quad (11)$$

and the τ_k are expressed similarly under condition of covariant differentiation with respect to the geometry of the second kind, i. e. if t_k, τ_k are Tchebyshev tensors of the first and second kind of the net ⁽⁸⁾ ⁽³⁾. Under this condition the bilinear forms $H_{kl} du^k \delta u^l, \chi_{kl} du^k \delta u^l$

$$H_{kl} = \tilde{a}^{rs} P_{rs} a_{kl} + \varepsilon^{rs} P_{rs} \varepsilon_{kl} \quad (I)$$

$$\chi_{kl} = \tilde{a}^{rs} \Pi_{rs} a_{kl} + \varepsilon^{rs} \Pi_{rs} \varepsilon_{kl} \quad (II)$$

are invariants of weight zero with respect to the change of normalization.

If we denote by u, v such curvilinear coordinates in which the equation (8) has the form $a_{12} du dv = 0$, then the forms (I) and (II) will take the following shape

$$H_{kl} du^k \delta u^l = k du \delta v + h \delta u dv \quad (12)$$

$$\chi_{kl} du^k \delta u^l = x du \delta v + \chi \delta u dv \quad (13)$$

where k and h are point invariants, and x and χ tangential invariants of Laplace-Darbeaux of the given conjugated net.

Consider the invariant forms

$$H_{[kl]} du^k \delta u^l = (H_{kl} - H_{lk}) du^k \delta u^l = (k - h) (du \delta v - \delta u dv) \quad (III)$$

$$\chi_{[kl]} du^k \delta u^l = (\chi_{kl} - \chi_{lk}) du^k \delta u^l = (x - \chi) (du \delta v - \delta u dv) \quad (IV)$$

$$\Omega_{kl} du^k \delta u^l = (\chi_{(kl)} - H_{(kl)}) du^k \delta u^l = 2(\chi + \eta - h - k) du dv \quad (V)$$

It is possible to find more convenient expressions for the coefficients of these forms, if we take into consideration the vector

$$\varphi_i = \tau_i - t_i + T_i = \bar{t}_i - t_i \quad (14)$$

where t_i and τ_i are still Tchebyshev tensors of the conjugated net, $T_i = (\Gamma_{\alpha i}^\alpha - G_{\alpha i}^\alpha)$ is equal to the Tchebyshev tensor of the first kind of the asymptotical net

$$b_{ij} du^i du^j = b_{12} dx dy = 0 \quad (15)$$

and determines the situation of the polar of the normalizing straight line of the first kind with respect to the tangential surface of S. Lie, and \bar{t}_i is the Tchebyshev tensor of the first kind of the conjugated net

$$\bar{a}_{ij} du^i du^j = 0 \quad (16)$$

which is apolar to the given one, i. e. satisfies the condition

$$\tilde{a}^{ij} \bar{a}_{ij} = 0 \quad (17)$$

Introducing the projective linear element of the surface (*)

$$ds = \frac{b_{\lambda\mu} du^\lambda du^\mu du^\nu}{b_{\alpha\beta} du^\alpha du^\beta} \quad (18)$$

we may show that

$$\varphi_i = \tilde{a}^{rs} \tilde{b}^{kl} b_{krs} a_{li} \quad (19)$$

whence follows that the vector φ_i is not only an absolute invariant of the change of normalization, but is also preserved under a projective bending of the surface, if it admits of such a bending. The vector

$$\psi_i = \varepsilon_{ir} \tilde{a}^{rs} \varphi_s \quad (20)$$

possesses evidently the same properties.

Consider besides the Riemannian metric with the linear element $ds^2 = b_{ij} du^i du^j$, where b_{ij} is the tensor of the asymptotical net with an arbitrary norming, and let us find the Tchebyshev tensor θ_i of the conjugated net with respect to the same metric. The expressions

$$\Phi = \varepsilon^{sr} \partial_s \varphi_r, \quad \Psi = \varepsilon^{sr} \partial_s \psi_r, \quad \theta = 2\varepsilon^{sr} \partial_s \theta_r \quad (21)$$

will be all invariants of the change of normalization and remain unchanged under a projective deformation. The coefficients of the forms (III), (IV) and (V) are expressed through them in the following form

$$H_{[kl]} = \frac{1}{2} [\theta - \Phi] \varepsilon_{kl}, \quad \chi_{[kl]} = \frac{1}{2} [\theta + \Phi] \varepsilon_{kl}, \quad \Omega_{kl} = \Psi a_{kl} \quad (22)$$

whence follows also their invariance under a projective deformation. If the asymptotical lines are taken for the parametric lines and $x = \text{const}$, $y = \text{const}$, the projective element of the surface has the form

$$ds = \frac{\beta dx^2 + \gamma dy^2}{2dx dy}$$

and the conjugated net is defined by the equation

$$e^{\rho} dx^2 + e^{-\rho} dy^2 = 0 \quad (23)$$

then

$$\theta = 2 \frac{\partial^2 \rho}{\partial x \partial y}, \quad \Phi = \partial_y (e^{2\rho} \gamma) - \partial_x (e^{-2\rho} \beta), \quad \Psi = \partial_x (e^{\rho} \gamma) + \partial_y (e^{-\rho} \beta) \quad (24)$$

The condition

$$\Phi + \theta = 0 \quad (25)$$

characterizes the net of equal point invariants; the condition

$$\Phi - \theta = 0 \quad (26)$$

the net of equal tangential invariants; the condition

$$\theta = 0 \quad (27)$$

the isometrically asymptotical net, and they all are satisfied for the so-called Jonas' nets which exist on surfaces of a special kind characterized in special asymptotical parameters by the condition

$$\partial \gamma / \partial y = \partial \beta / \partial x \quad (28)$$

The condition

$$\Psi = 0 \quad (29)$$

characterizes the nets which shall be called stratifying. In order that a net should be stratifying it is necessary and sufficient that its axes and Green's rays should form a pair of congruences stratifiable in the direction of the ray. The invariants of Laplace-Darbeaux of such a net stand in the relation $h + k = \chi + \kappa$.

On every surface there are infinitely many stratifying nets and in the asymptotic coordinates they are expressed by the equation (23) under the condition

$$\frac{\partial}{\partial x}(e^2\gamma) + \frac{\partial}{\partial y}(e^{-2}\beta) = 0 \quad (30)$$

If the conditions (27) and (29) are satisfied simultaneously, we have a net R which is thus a stratifying isometrically conjugated net. On a surface of the second order every conjugated net is stratifying. A net satisfying the condition

$$\Phi = 0 \quad (31)$$

is characterized by the property that if its equation has the form (23), then the net

$$e^{-2}\beta^2 dx^2 - e^{2\gamma}\gamma^2 dy^2 = 0 \quad (32)$$

will be stratifying. In an arbitrary system of coordinates the tensor of the net (32) has the following form

$$c_{ij} = b_{ijk} e^{kl} \varphi_l \quad (33)$$

On Jonas' surface characterized by the condition (28) the net

$$\beta^2 dx^2 - \gamma^2 dy^2 = 0 \quad (34)$$

will be stratifying.

If a surface in the Euclidean space is normalized by means of its usual normal and the infinitely removed straight line of the tangential plane, then the geometry of the first kind coincides with the inner geometry of the surface in the sense of Gauss^(*), and the geometry of the second kind, with the geometry determined by the linear element of the spherical representation $d\sigma^2 = c_{ij} du^i du^j$, and the coefficients of the invariant forms (I) and (II) assume the form

$$H_{kl} = \frac{1}{2} [\tilde{a}^{rs} (\nabla_s t_r - t_r t_s) a_{kl} + \varepsilon^{rs} \nabla_s t_r \varepsilon_{kl}]$$

$$Z_{kl} = \frac{1}{2} [\tilde{a}^{rs} (\nabla_s \tau_r - \tau_r \tau_s - c_{rs}) a_{kl} + \varepsilon^{rs} \nabla_s \tau_r \varepsilon_{kl}]$$

In this case

$$\varphi_i = \tau_i - t_i + \frac{\partial \lg k}{\partial u^i} \quad (35)$$

where K is the Gaussian curvature of the surface, and

$$b_{ij} = \frac{\tau_i + t_j}{2} \quad (36)$$

Received
5. VII. 1946.

REFERENCES

- ^{*} A. Norden, C. R. Acad. Sci. URSS, 49, No. 8 (1945). ^{*} E. Lano, Projective Diff. Geometry of Curves and Surfaces, p. 96, 1932. ^{*} A. Norden, C. R., 192, 135 (1930). ^{*} G. Fubini, E. Čech, Introduction à la géométrie projective différentielle, p. 64, 1931. ^{*} A. Norden, Тр. семинара по векторн. и тензорн. анализу, II—III, 264 (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

MATHEMATICS

ON INTERSECTION OF OPEN SUBSETS IN THE PRODUCT
OF TOPOLOGICAL SPACES

By N. A. SHANIN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 14. II. 1946)

§ 1. General Remarks. In the present note the terminology and notations of (1) are used.

Topological space will be called simply T -space. In general, only the following conditions will be imposed on the T -space: the sum of any system and the intersection of a finite number of open sets are open sets, the space itself and the void set are open sets. The minimum of the powers of open bases in a T -space R will be called weight of the T -space R . The minimum of the powers of the dense subsets of a T -space R is called pseudo-weight. p will denote the function which with every T -space R associates its weight $p(R)$; q will denote the function which associates with every T -space R its pseudo-weight $q(R)$. Note that $q(R) \leq p(R)$ for every T -space R .

Let X be a function defined on a set Ξ , which associates with every element $\xi \in \Xi$ a set $X(\xi)$. The product of all the sets $X(\xi)$ (*i. e.* the set of all functions x defined on Ξ such that $x(\xi) \in X(\xi)$ for every $\xi \in \Xi$) will be denoted by X^Ξ . If all the values of the function X are identical, $X(\xi) = R$ for every $\xi \in \Xi$; the product X^Ξ will also be denoted by R^Ξ (« R to the Ξ -th power»).

A subset K of the product X^Ξ will be called cylindrical subset if there exist $\xi \in \Xi$ and $A \subset X(\xi)$ such that K is the set of all the elements of the product X^Ξ whose projections in $X(\xi)$ belong to A . If K is neither void set nor X^Ξ itself, then ξ and A in the definition of K are unique. The set A will be called base of the cylindrical set K .

If for all $\xi \in \Xi$ the values $X(\xi)$ of the function X are T -spaces, and open sets are defined in X^Ξ by Tychonoff's method⁽²⁾, then the T -space X^Ξ is called the product of the T -spaces $X(\xi)$. If all the values of X coincide (*i. e.* they equal the same topological space): $X(\xi) = R$ for all $\xi \in \Xi$, then the T -space R^Ξ is called «the T -space R to the Ξ -th power». A set U will be called elementary open subset of the product X^Ξ , if U can be represented as intersection of a finite number of cylindrical sets whose bases are open subsets of the factors of the product. Let U be an elementary open subset of the product X^Ξ . The set of the $\xi \in \Xi$ for which the projection of the elementary open subset U in the T -space $X(\xi)$ is not equal to $X(\xi)$, will be called basis of the set U .

§ 2. Calibres and Universal Calibres of Topological Spaces. We say that a family \mathfrak{A}^* is a kernel of another family \mathfrak{A} if $\mathfrak{A}^* \subset \mathfrak{A}$, $\mathfrak{A}^* \sim \mathfrak{A}$ and $\bigcap \mathfrak{A}^* \neq \Delta$. We say that the family \mathfrak{A} possesses a kernel if there is a family \mathfrak{A}^* which is kernel of \mathfrak{A} . Let \mathfrak{M} be a family of non-void sets. We say that a cardinal number m is a calibre of the family \mathfrak{M} if $m > 1$ and every part of \mathfrak{M} whose power is m possesses a kernel. The calibre m is called realizable calibre if $m \leq \mathfrak{M}$.

In case $m > \mathfrak{M}$, m will be called non-realizable calibre of the family \mathfrak{M} .

Definition 1. We say that a cardinal number m is a calibre (a realizable calibre, a non-realizable calibre) of a T -space R , if m is a calibre (resp. a realizable calibre, a non-realizable calibre) of the family of all non-void open subsets of the T -sets R .

It is evident that \aleph_0 is the minimal cardinal number which may be the calibre of an infinite T -space.

Theorem 1. Every cardinal number m satisfying the condition $\chi(m) > q(R)$ is a calibre of a T -space R .

Remark. Between two calibres of a T -space R cardinal numbers may be contained that are no calibres of R . If m is a calibre of a T -space R , then a cardinal number less (or greater) than m is not necessarily calibre of R .

Theorem 2. If a T -space R is a continuous image of a T -space S and the cardinal number m is a calibre of S , then m is a calibre of T -space R .

Let \mathfrak{M} be a family of non-void sets. We say that the cardinal number m is the universal calibre of the family \mathfrak{M} , if $m > 1$ and the following condition is fulfilled: whatever be a set A of power m and a function Γ on the set A with values $\Gamma(x)$ belonging to \mathfrak{M} for every $x \in A$, there exists a set A^* such that $A^* \subset A$, $A^* \sim A$ and $\bigcap_{x \in A^*} \Gamma(x) \neq \Delta$.

Definition 2. We say that a cardinal number m is the universal calibre of a T -space R if m is the universal calibre of the family of all non-void open subsets of R .

Lemma. (a) A cardinal number m is the universal calibre of the family \mathfrak{M} of non-void sets if and only if the cardinal numbers m and $\chi(m)$ are calibres of the family \mathfrak{M} . (b) A cardinal number m is the universal calibre of a T -space R if and only if the cardinal numbers m and $\chi(m)$ are calibres of R .

Remark. If the cardinal number m is not regular (i. e. if $\chi(m) < (m)$), then it may happen that m will be a calibre, but not universal, of a T -space R .

Lemma. Let \mathfrak{U} be an open basis of a T -space R containing no void set. A cardinal number m is the universal calibre of R if and only if m is the universal calibre of \mathfrak{U} .

§ 3. Calibres of the Product of Topological Spaces. In the following Theorems 3—11 we will suppose that a non-void set Ξ and a function X on Ξ are given; the latter associates with every $\xi \in \Xi$ a non-void T -space $X(\xi)$.

Theorem 3. Suppose that an infinite cardinal number m satisfies the following conditions: (1) for every $\xi \in \Xi$ m is the universal calibre of the T -space $X(\xi)$ (in other words, both m and $\chi(m)$ are calibres of $X(\xi)$); (2) if the set Ξ is infinite, then $\chi(m) > \aleph_0$; (3) at least one of the following conditions (a), (b) is fulfilled: (a) m is a regular cardinal number; (b) for every cardinal number $a < m$ there is a cardinal number c such that $a < c < m$ and $J(c) < \chi(m)$, where $J(c)$ is the set of all $\xi \in \Xi$ for which c is no universal calibre of the T -space $X(\xi)$. Then m is a calibre (even the universal calibre) of the product X^Ξ .

Remark. If $\chi(m) > \Xi$, then for every cardinal number c the inequality $J(c) < \chi(m)$ holds. Thus, the condition (3) of Theorem 3 is fulfilled, in particular, in the case where $\chi(m) > \Xi$.

Proof of Theorem 3 is based upon a theorem from (1). Let us denote by \mathfrak{U} the family of all non-void elementary open subsets of the product X^Ξ . It is sufficient to show that the cardinal number m is the universal calibre of the family \mathfrak{U} . Let A be a set of power m and let Γ be a function on A associating with every $\alpha \in A$ an element $\Gamma(\alpha)$ of \mathfrak{U} . Denote

by Φ the function associating with every $\alpha \in A$ a finite or void subset $\Phi(\alpha)$ of Ξ which is the basis of the set $\Gamma(\alpha)$. It is easy to see that there exists an integer $n \geq 0$ and a set A_1 such that $A_1 \subset A$, $A_1 \sim A$ and for every $\alpha \in A_1$ the set $\Phi(\alpha)$ consists of n elements. If $n=0$, then $\bigcap \Gamma(\alpha) = X^\Xi \neq \Delta$. If $n \geq 1$, then we apply the theorem from⁽¹⁾ to the case $H=A_1$. By this theorem there is a subset H^* of H equivalent to H , and three functions Φ_1, Φ_2, Φ_3 everyone of which maps H^* into the system of void and finite subsets of Ξ in such a way that the conditions (1)–(7) of this theorem are fulfilled. The properties of H^* and Φ_1, Φ_2, Φ_3 make clear the plan of completing the proof of existence of the set A^* with the properties: $A^* \subset H^*$, $A^* \sim A$, $\bigcap_{\alpha \in A^*} \Gamma(\alpha) \neq \Delta$.

The following theorems are corollaries of Theorem 3.

Theorem 4. *Let m be an infinite regular cardinal number satisfying the following two conditions: (1) for every $\xi \in \Xi$ the cardinal number m is a calibre of the T -space $X(\xi)$; (2) if Ξ is infinite, m is non-enumerable. Then m is a calibre (even the universal calibre) of the product X^Ξ .*

Theorem 5. *Let m be a cardinal number satisfying the following conditions: (1) $\chi(m) > q(X(\xi))$ for each $\xi \in \Xi$; (2) $\chi(m) > \aleph_0$. Then m is a calibre (even the universal calibre) of the product X^Ξ .*

Theorem 6. *If $q(X(\xi)) \leq \aleph_0$ for each $\xi \in \Xi$, then every cardinal number m satisfying the condition $\chi(m) > \aleph_0$ is a calibre (even the universal calibre) of the product X^Ξ .*

Theorem 7. *Suppose that $q(X(\xi)) \leq \aleph_0$ for each $\xi \in \Xi$ and \mathfrak{B} is a non-enumerable family of non-void open subsets of the product X^Ξ . Then for every cardinal number $\alpha < \mathfrak{B}$ there exists a family \mathfrak{B} possessing the properties: $\mathfrak{B} \subset \mathfrak{B}$, $\mathfrak{B} > \alpha$, $\bigcap \mathfrak{B} \neq \Delta$.*

The conditions (1), (2), (3) of Theorem 3 are sufficient, but, in general, not necessary for the cardinal number m to be a calibre of the product X^Ξ . Theorems 8–11 deal with necessary conditions.

Theorem 8. *A non-enumerable regular cardinal number m is a calibre of the product X^Ξ if and only if m is a calibre of every T -space $X(\xi)$.*

Theorem 9. *Let R and S be T -spaces. If an infinite cardinal number m is a calibre of the product $R \times S$ and m does not exceed the power of the family of all open subsets of S , then m is a calibre of the T -space R .*

Theorem 10. *Suppose that for each $\xi \in \Xi$ $X(\xi)$ is a T_2 -space consisting of at least two points, and let m be a calibre of the product X^Ξ satisfying the condition $m \leq 2^\Xi$. Then m is a universal calibre of $X(\xi)$ for each $\xi \in \Xi$ and $\chi(m) > \aleph_0$.*

Theorem 11. *Let Ξ be an infinite set and let $X(\xi)$ be a T_2 -space of finite or enumerable weight consisting of at least two points for each $\xi \in \Xi$. A cardinal number m is a realizable calibre of the product X^Ξ if and only if $\aleph_0 < \chi(m) \leq m \leq 2^\Xi$. A cardinal number m is a non-realizable calibre of the product X^Ξ if and only if $m > 2^\Xi$.*

Received
14. II. 1946.

REFERENCES

- ¹ N. A. Shanin, C. R. Acad. Sci. URSS, 53, No. 5 (1946). ² A. Tychonoff, Math. Ann., 102, 544 (1929).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

THEORY OF ELASTICITY

**A GENERALIZATION OF THE METHOD OF COMPLEX
DISPLACEMENTS TO THE NON-HOMOGENEOUS PROBLEM
OF THE THEORY OF SHELLS**

By V. V. NOVOZHILOV

(Communicated by V. L. Posdunine, Member of the Academy, 20. III. 1946)

1. In ⁽¹⁾ we have established that by introducing three auxiliary functions called «complex stresses» the order of the differential equations in the theory of thin shells can be reduced by half. A similar result is proved there also for three other auxiliary functions called «complex displacements». However, while the equations in «complex stresses» were deduced on the assumption that there is a surface load on the shell, the equations in «complex displacements» were given only for the homogeneous problem.

Below we remove this restriction and the equations in complex displacements will be extended also to the case of non-homogeneous problem.

2. If the Poisson coefficient $\mu = 0$, the complex representation of the equations of the theory of thin shells is possible without neglecting anything in the equilibrium equations of the shell element and the continuity equations of its mean surface ⁽¹⁾. For these reasons we shall in the sequel assume that $\mu = 0$. This will enable us to obtain absolutely coordinated results which will be then at the cost of certain discards generalized to arbitrary values of the Poisson coefficient.

3. Suppose that the complex stresses $\tilde{T}_1, \tilde{T}_2, \tilde{S}$ are determined. Then the following two systems of differential equations may be written for the displacements u, v, w of the shell:

$$\begin{aligned} E \delta z_1 &= \frac{1}{2} (\tilde{T}_1 + \bar{T}_1) \\ E \delta z_2 &= \frac{1}{2} (\tilde{T}_2 + \bar{T}_2) \end{aligned} \quad (1)$$

$$\begin{aligned} E \delta \frac{w}{2} &= \frac{1}{2} (\tilde{S} + \bar{S}) \\ -ic_0 E \delta x_1 &= \frac{1}{2} (\tilde{T}_2 - \bar{T}_2) \\ -ic_0 E \delta x_2 &= \frac{1}{2} (\tilde{T}_1 - \bar{T}_1) \end{aligned} \quad (2)$$

$$ic_0 E \delta \tau = \frac{1}{2} (\tilde{S} - \bar{S})$$

where $\bar{T}_1, \bar{T}_2, \bar{S}$ are the conjugated complex stresses, and $c_0 = \delta / \sqrt{12}$.

Put in these equations $\tilde{T}_1, \tilde{T}_2, \tilde{S}$ equal to zero and denote the solutions of the system (1) in this case by $1/2 \tilde{u}', 1/2 \tilde{v}', 1/2 \tilde{w}'$, and the solutions of the system (2) by $1/2 \tilde{u}'', 1/2 \tilde{v}'', 1/2 \tilde{w}''$.

Observe that the real parts of $\tilde{u}', \tilde{v}', \tilde{w}'$ and $\tilde{u}'', \tilde{v}'', \tilde{w}''$ will be respectively the same and equal to the displacements u, v, w . This latter assertion is guaranteed by the fact that the complex stresses $\tilde{T}_1, \tilde{T}_2, \tilde{S}$

are solutions of the equations (12) of⁽¹⁾, which combine in themselves the equilibrium conditions of the shell element as well as the continuity condition of its deformation. For the coincidence of the imaginary parts of $\tilde{u}', \tilde{v}', \tilde{w}'$ and $\tilde{u}'', \tilde{v}'', \tilde{w}''$ there is, however, no guarantee.

If we yet assume that $\tilde{u}' = \tilde{u}'', \tilde{v}' = \tilde{v}'', \tilde{w}' = \tilde{w}''$, then it will lead us to the differential equations

$$\begin{aligned}\tilde{\varepsilon}_1 &= -ic_0 \tilde{x}_2 \\ \tilde{\varepsilon}_2 &= -ic_0 \tilde{x}_1 \\ \frac{1}{2} \tilde{\omega} &= ic_0 \tilde{\tau}\end{aligned}\quad (3)$$

obtained in⁽¹⁾, in which the free terms are absent.

Hence we may conclude that the condition of coincidence of $\tilde{u}', \tilde{u}''; \tilde{v}', \tilde{v}''; \tilde{w}', \tilde{w}''$ consists in the homogeneity of the problem.

If the problem is non-homogeneous, then the indicated coincidence will not have place, but we may put

$$\begin{aligned}\tilde{u} &= \tilde{u}' = \tilde{u}'' + i\varphi_1 \\ \tilde{v} &= \tilde{v}' = \tilde{v}'' + i\varphi_2 \\ \tilde{w} &= \tilde{w}' = \tilde{w}'' + i\varphi_3\end{aligned}\quad (4)$$

where $\varphi_1, \varphi_2, \varphi_3$ are real functions of the surface load, vanishing if $q_1 = q_2 = q_n = 0$.

In virtue of (4) and the definition of $\tilde{u}', \tilde{u}'', \tilde{v}', \tilde{v}'', \tilde{w}', \tilde{w}''$ given above, we shall have

$$\begin{aligned}E \tilde{\varepsilon}_1 &= \tilde{T}_1, & -ic_0 E \tilde{\varepsilon}_2 &= \tilde{T}_2 - T_2^0 \\ E \tilde{\varepsilon}_2 &= \tilde{T}_2, & -ic_0 E \tilde{\varepsilon}_1 &= \tilde{T}_1 - T_1^0 \\ E \tilde{\omega} &= \tilde{S}, & ic_0 E \tilde{\tau} &= \tilde{S} - S^0\end{aligned}\quad (5)$$

where T_1^0, T_2^0, S^0 are some other real functions of the surface load. We proceed to determine these functions.

4. Let us write down one of Goldenweiser's identities⁽⁴⁾

$$\begin{aligned}\frac{\partial A_2 \tilde{x}_2}{\partial a_1} - \frac{\partial A_1 \tilde{x}_1}{\partial a_2} - \frac{\partial A_1 \tilde{\tau}}{\partial a_1} - \frac{\partial A_2 \tilde{\tau}}{\partial a_2} + \frac{1}{R_2} \frac{\partial A_1}{\partial a_2} \tilde{\omega} - \\ - \frac{1}{R_1} \left(\frac{\partial A_2 \tilde{\varepsilon}_1}{\partial a_1} - \frac{\partial A_1 \tilde{\varepsilon}_2}{\partial a_2} - \frac{\partial A_1 \tilde{\omega}}{\partial a_1} \right) = 0\end{aligned}\quad (6)$$

If we introduce here instead of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{\omega}, \tilde{x}_1, \tilde{x}_2, \tilde{\tau}$ their expressions according to (5), we obtain the following differential relation between the complex stresses:

$$\begin{aligned}\frac{1}{A_1 A_2} \left[\frac{\partial A_2 \tilde{T}_1}{\partial a_1} + \frac{\partial A_1 \tilde{S}}{\partial a_2} + \frac{\partial A_1 \tilde{S}}{\partial a_1} - \frac{\partial A_2 \tilde{T}_2}{\partial a_1} \right] + \\ + i \frac{c_0}{A_1 A_2} \left[\frac{1}{R_1} \frac{\partial A_2 \tilde{T}_2}{\partial a_1} - \frac{1}{R_1} \frac{\partial A_2 \tilde{T}_1}{\partial a_2} - \frac{2}{R_1} \frac{\partial A_1 \tilde{S}}{\partial a_2} - \frac{2}{R_2} \frac{\partial A_1 \tilde{S}}{\partial a_1} \right] = \\ = \frac{1}{A_1 A_2} \left[\frac{\partial A_2 T_1^0}{\partial a_1} + \frac{\partial A_1 S^0}{\partial a_2} + \frac{\partial A_1 S^0}{\partial a_1} - \frac{\partial A_2 T_2^0}{\partial a_1} \right]\end{aligned}\quad (7)$$

It may be represented also in the form

$$\begin{aligned}\frac{1}{A_1 A_2} \left[\frac{\partial A_2 \tilde{T}_1}{\partial a_1} + \frac{\partial A_1 \tilde{S}}{\partial a_2} + \frac{\partial A_1 \tilde{S}}{\partial a_1} - \frac{\partial A_2 \tilde{T}_2}{\partial a_1} \right] + \\ + i \frac{c_0}{1 - i \frac{c_0}{R_1}} \left[\frac{1}{R_1 A_1} \frac{\partial \tilde{T}_1}{\partial a_1} - \frac{1}{R_1 A_2} \frac{\partial \tilde{S}}{\partial a_2} - \frac{2}{R_2} \frac{1}{A_1 A_2} \frac{\partial A_1 \tilde{S}}{\partial a_1} \right] = \\ = \frac{1}{1 - i \frac{c_0}{R_1}} \frac{1}{A_1 A_2} \left[\frac{\partial A_2 T_1^0}{\partial a_1} + \frac{\partial A_1 S^0}{\partial a_2} + \frac{\partial A_1 S^0}{\partial a_1} - \frac{\partial A_2 T_2^0}{\partial a_1} \right]\end{aligned}\quad (8)$$

Comparing this expression with the first of the equations of the system (12) in ⁽¹⁾, we find that the left-hand side of (8) precisely coincides with the left-hand side of the equation mentioned. But then their right-hand sides must coincide also, whence

$$\frac{1}{A_1 A_2} \left[\frac{\partial A_2 T_1^0}{\partial \alpha_1} + \frac{\partial A_1 S^0}{\partial \alpha_2} + \frac{\partial A_1}{\partial \alpha_2} S^0 - \frac{\partial A_2}{\partial \alpha_1} T_2^0 \right] + q_1 = 0 \quad (9)$$

Similar considerations regarding two other identities of Goldenweiser yield another two differential relations between T_1^0 , T_2^0 , S^0 which together with (9) form the differential system

$$\begin{aligned} \frac{1}{A_1 A_2} \left[\frac{\partial A_2 T_1^0}{\partial \alpha_1} + \frac{\partial A_1 S^0}{\partial \alpha_2} + \frac{\partial A_1}{\partial \alpha_2} S^0 - \frac{\partial A_2}{\partial \alpha_1} T_2^0 \right] + q_1 &= 0 \\ \frac{1}{A_1 A_2} \left[\frac{\partial A_2 S^0}{\partial \alpha_1} + \frac{\partial A_1 T_2^0}{\partial \alpha_2} + \frac{\partial A_2}{\partial \alpha_1} S^0 - \frac{\partial A_1}{\partial \alpha_2} T_1^0 \right] + q_2 &= 0 \\ \frac{T_1^0}{R_1} + \frac{T_2^0}{R_2} &= q_n \end{aligned} \quad (10)$$

where, according to the preceding considerations, T_1^0 , T_2^0 , S^0 must be determined as the partial solutions of this system, vanishing for $q_1 = q_2 = q_n = 0$.

But the equations (10) are precisely the system of the momentless theory and, consequently, T_1^0 , T_2^0 , S^0 in the formulae (5) are the partial solutions of the momentless theory for the shell under consideration and the given surface load.

5. Eliminate now from the equations (5) the complex stresses. Then we obtain the differential system

$$\begin{aligned} \tilde{\varepsilon}_1 + ic_0 \tilde{x}_2 &= T_1^0 / E\delta \\ \tilde{\varepsilon}_2 + ic_0 \tilde{x}_1 &= T_2^0 / E\delta \\ \frac{1}{2} \tilde{\omega} - ic_0 \tilde{\tau} &= S^0 / E\delta \end{aligned} \quad (11)$$

which is the required generalization of the system (20) of ⁽¹⁾ to the case of non-homogeneous problem. The real parts of the functions \tilde{u} , \tilde{v} , \tilde{w} determined as the general solutions of the given system will be equal to the general expressions for the displacements u , v , w . The meaning of the free terms of the system (11) has been established above.

It may be noted that in the majority of cases the determination of these terms will not be difficult, since for many shells the momentless problem may be solved by sufficiently simple means.

6. It remains to generalize the obtained results to the case of arbitrary values of the Poisson coefficient. As we have pointed out in ⁽¹⁾, such a generalization may be achieved only at the cost of a series of discards in the equilibrium equations and the equations of continuity. However, the error in these equations will not in the majority of practical problems exceed the errors of Kirchhoff's hypotheses. Arguing on similar lines as in the paper referred to above, we arrive at the following differential system:

$$\begin{aligned} \frac{\tilde{\varepsilon}_1 + \mu \tilde{\varepsilon}_2}{1 - \mu^2} + ic \tilde{x}_2 &= \frac{T_1^0}{E\delta} \\ \frac{\tilde{\varepsilon}_2 + \mu \tilde{\varepsilon}_1}{1 - \mu^2} + ic \tilde{x}_1 &= \frac{T_2^0}{E\delta} \\ \frac{\tilde{\omega}}{2(1 + \mu)} - ic \tilde{\tau} &= \frac{S^0}{E\delta} \end{aligned} \quad (12)$$

where $c = \delta / \sqrt{12(1 - \mu^2)}$ which is the required generalization of the system (25) of ⁽¹⁾. The functions of the surface load T_1^0 , T_2^0 , S^0 are determined in this case in the same manner as in the system (11).

7. In conclusion we observe that from sect. 4 follows that by T_1^0, T_2^0, S^0 we may understand not only partial solutions of the momentless theory, but also its general solutions, since then too formulae (5) remain coordinated with the equations for complex stresses. Hereby in the right-hand sides of the differential systems (11) and (12) there appear arbitrary quantities which can well be dispensed with. All the same, their introduction will on occasion be useful, since it enables us in some cases to simplify the computations.

A similar example is the «superfluous» function in the well-known general solution of Papkovich ⁽²⁾ of the equations of the theory of elasticity.

Received
20. III. 1946.

REFERENCES

- ¹ V. Novozhilov, Bull. Acad. Sci. URSS, série techn., No. 1 (1946). ² A. Goldenweiser, Applied Math. a. Mech., 4, No. 2 (1940). ³ И. П а п к о в и ч, Теория упругости, 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

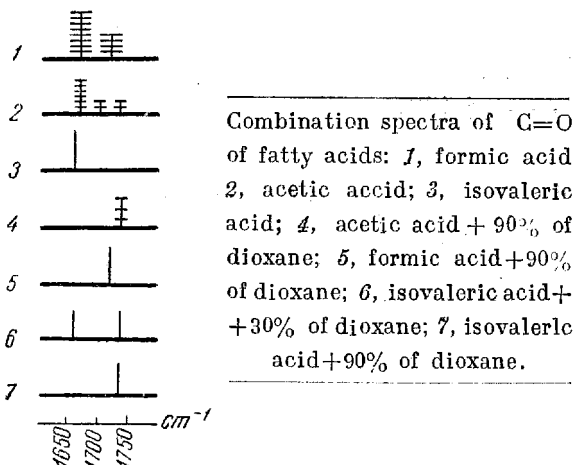
PHYSICS

**EFFECT OF THE HYDROGEN BOND ON THE FREQUENCIES
OF THE CARBONYL GROUPING IN THE FATTY ACIDS**

By M. I. BATUEV

(Communicated by A. N. Nesmeyanov, Member of the Academy, 6. VI. 1946)

In his previous papers ⁽¹⁾ the author reported data on the hydrogen bond in dimeric and polymeric complexes of the fatty acids, secured by studying changes in the frequency of the hydroxyl grouping in the combination spectra



of various acids when in a state of pure liquids or dissolved in acetone or dioxane. Since the other link involved in the formation of the hydrogen bond is the carbonyl grouping, C=O, there, naturally, comes up the question as to whether the peculiarities of the association of fatty acids, established by the author in studying the O—H frequency, may or may not be reflected in the C=O grouping as well.

In the combination spectra of ketones and esters the carbonyl grouping is characterized by a frequency ranging from 1700 cm⁻¹ up to 1770 cm⁻¹ ⁽²⁾. The frequencies of the carbonyl grouping of fatty acids have been found to lie far outside this range and to differ essentially from each other so as to fall into three groups. In the first group belongs formic acid, its carbonyl grouping being characterized not by a line but rather by a band about 90 cm⁻¹ wide. This band has two sharply expressed overlapping steps. One of them with its middle at 1670 cm⁻¹ is more intense than the other which centers about 1724 cm⁻¹. The second group includes acetic acid which in the region of C=O shows a triplet of three diffuse lines of which most intense is the one with the lowest frequency, 1668 cm⁻¹, the other two being weaker. The third group includes all other acids, investigated by the author, beginning with propionic acid. They show one intense reduced frequency of the carbonyl

grouping at about 1660 to 1670 cm^{-1} in the form of a narrow line, while their higher frequencies at about 1700 and 1740 cm^{-1} appear as vanishingly slight traces.

Thus, the lengthening and the branching of the carbon chain in the fatty acids leads, even in the case of acetic acid, to the dissolution of the wide

Table 1

C=O Frequency of the Fatty Acids
(the exciting line was 4358 Å ; b is a wide line; dbl is a double line)

Acid	Formula	C=O frequency, cm^{-1}	Character of frequency
I Formic	$\text{H}-\text{COOH}$	$1670(6b), 1724(3b)$	Band about 90 cm^{-1} wide*
II Acetic	CH_3-COOH	$1668(6), 1713$ $(2dbl), 1743(2)$	Three dif- fuse lines
III Propionic	$\text{CH}_3-\text{CH}_2-\text{COOH}$	$1670(6)$	Line
Butyric normal . .	$\text{CH}_3-\text{CH}_2-\text{CH}_2-\text{COOH}$	$1661(6)$	"
Isobutyric	$(\text{CH}_3)_2\text{CH}-\text{COOH}$	$1660(6)$	"
Isovaleric	$(\text{CH}_3)_2\text{CH}-\text{CH}_2-\text{COOH}$	$1660(6)$	"
Trimethylacetic**	$(\text{CH}_3)_3\text{C}-\text{COOH}$	$1656(6)$	"
Diethylacetic** . .	$(\text{C}_2\text{H}_5)_2\text{CH}-\text{COOH}$	$1658(6)$	"
Enanthic	$(\text{CH}_3)-(\text{CH}_2)_6-\text{COOH}$	$1663(6)$	"

* The band consists of two fused smaller bands of different intensity; the middles of the smaller bands are given here.

** The author wishes to express his thanks to A. L. Liberman for placing trimethyl- and diethylacetic acids at the author's disposal.

C=O band, observed in formic acid, into three distinct lines, the most intense of which is that with the lowest frequency, *viz.* 1668 cm^{-1} . With propionic acid and the higher members of the homologous series of fatty acids investigated by the author (Table 1) only one sharp line—of low frequency—lying in the region of 1660 to 1670 cm^{-1} was observed.

The spectra of solutions in dioxane showed a sharp alteration of the C=O frequencies (Table 2). Actually, the wide band remained in place on addition of 30 per cent of dioxane to formic acid but the distribution of intensities within it was essentially different from that within the C=O band of pure liquid formic acid: the intensity of the portion of the band with the higher frequency increased in relatively higher degree to become equal to the intensity of the low frequency portion of the band. On addition of 90 per cent of dioxane the low-frequency portion of the band disappeared, there being left but one intense narrow line of increased frequency, *viz.* 1730 cm^{-1} .

When 30 per cent of dioxane were added to acetic acid, the intensities of the lines in the triplet, observed in pure liquid acetic acid, were changed: the intensity of the frequency 1668 cm^{-1} was subjected to a relatively sharp drop, that of the frequency 1743 cm^{-1} sharply increased, as well as that of 1713 cm^{-1} . In the event of further decrease in concentration there remained only one line, that of 1740 cm^{-1} (on addition of 90 per cent of dioxane).

When 60 per cent of dioxane were added to the acids of group III—isobutyric, isovaleric, two lines of equal intensity— 1660 and 1740 cm^{-1} —were observed in the spectra instead of one line 1660 cm^{-1} . On adding 60 per cent of dioxane the 1660 cm^{-1} line disappeared almost completely, very feeble traces of that line being left, whereas the relatively intense and narrow line 1740 cm^{-1} became very prominent.

As shown by the author in previous papers (¹), two kinds of association through hydrogen bond are possible in the fatty acids: dimeric and polymeric. The former predominates in acids with long and branched carbon

Table 2

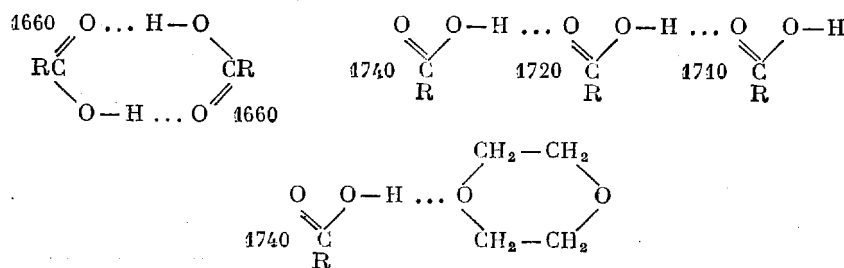
Alteration of the C=O Frequency on Dissolving of the Acids in Dioxane

	C=O frequency, cm ⁻¹	Character of frequency
Formic acid 100%	1670(6b), 1724(3b)	Band about 90 cm ⁻¹ wide*
» » +30% dioxane	1670(4b), 1724(4b)	Same**
» » +90% »	1730 (4)	Narrow line
Acetic acid 100%	1668(6), 1713(2b), 1743 (2b)	Three diffuse lines
» » +30% dioxane	1668(2), 1712(2b), 1743 (4b)	Same
» » +90% »	1740 (4b)	Line
Isovaleric acid	1660 (6)	Narrow line
» » +30% dioxane	1660(3), 1740(4)	Two lines
» » +60% dioxane	1740 (4)	Narrow line

* The band consists of two fused smaller bands of different intensity; the middles of the smaller bands are given here.

** Two fused bands of uniform intensity.

chain, beginning with propionic acid; in acetic and, to an even greater degree, in formic acid polymeric forms, originated through hydrogen bond, are capable of existing along with dimeric forms. The carbonyl groupings in these complexes as well as in those formed through hydrogen bond between the molecules of the acids and of dioxane are found to be dissimilar:



In the cyclic dimer the distances C=O, O—H and O...H are relatively stable. Therefore any considerable diffusiveness is not to be expected either in the frequency of the hydroxyl grouping (¹) or in that of the carbonyl grouping. In the polymeric complexes there exist two different carbonyl groups: that at the end of the polymeric chain, free from a hydrogen bond, and the other one, participating in the formation of the polymeric hydrogen bond; the fluctuations in the latter C=O frequency as well as in that of O—H in the polymeric chain are much less restricted than in the cyclic dimer; for that reason this grouping may be the cause of certain diffusiveness of the C=O frequency, whereas that C=O grouping which is free from a hydrogen bond must, naturally, cause the appearance in the spectra of a narrow C=O line. In the complexes, formed through the hydrogen bond between the molecules of the acid and dioxane, there is only one carbonyl grouping, free from hydrogen bond, which, as was pointed out, may be represented in the form of a usual line with the respective frequency.

In agreement with the above as well as with the results, obtained by the author while investigating the O—H band (¹), one may draw the following conclusions concerning the C=O frequencies of the fatty acids in the pure liquid state and in dioxane solutions, as found by the authors.

The 1660 cm⁻¹ frequency observed in acids, beginning with propionic acid, characterizes, obviously, the C=O frequency of the dimeric ring; the association in the form of the dimeric ring doubtlessly predominates in these acids. This frequency is also present in the spectra of acetic and formic acids, thus pointing at the existence of the cyclic dimers in these acids, too.

The 1740 cm⁻¹ frequency is observed in the complexes formed between acid and dioxane; it is characteristic of the frequency of a carbonyl grouping free from a hydrogen bond. The appearance of this frequency alone in the weak dioxane solutions (10 to 15 per cent) of acids of all three groups shows that the dimeric and polymeric complexes, formed by the molecules of the acids, are in these solutions completely decomposed.

The ~1660 and ~1740 cm⁻¹ frequencies are observed in the spectra of acetic acid; besides, in the spectra of this acid there is observed a frequency of 1713 cm⁻¹. Whereas the first of the three frequencies indicates the presence of dimers in the pure liquid acetic acid, the latter two frequencies give evidence in favour of the presence in it of the polymeric forms; the 1713 cm⁻¹ frequency in acetic acid is somewhat diffused and is of the nature of a double line.

In formic acid the observable C=O band covers all the three above-mentioned frequencies, thus indicating the presence therein of both dimers and of a high content of polymers. When dissolved in dioxane, these dimers and polymers are decomposed, and in a 10 per cent solution of formic acid in dioxane they are decomposed almost completely, as evidenced by a sharply distinct single narrow line 1730 cm⁻¹ that refers to the frequency of C=O, free from a hydrogen bond.

In the physical-chemical literature acetic, as well as formic, acids are generally supposed to form dimers. This, however, has been checked experimentally only for vapours of these acids and for their weak solutions in neutral solvents. There is no evidence that the same should be true of the liquid state of the two above-mentioned acids (²). Now it is in the liquid state that the author has found these acids to form polymers as well as dimers.

Thus, we arrive at the conclusion that not only the O—H frequency but the C=O frequency as well is subject in a great degree to the influence of intermolecular forces and like the former provides an excellent means for investigation of hydrogen bonds.

The author wishes to express his thanks to G. S. Landsberg, Corresponding Member of the Academy, for discussion of the results obtained.

Institute of Organic Chemistry,
Academy of Sciences of the USSR.

Received
6. VI. 1946.

REFERENCES

- ¹ M. I. Batuev, C. R. Acad. Sci. URSS, 52, No. 5 (1946); 53, No. 4 (1946)
- ² G. V. L. N. Murty, T. R. Secharidi, Proc. Indian Acad. Sci., A, 15, 238 (1942)

PHYSICS

IONIZATION OF GASES AND THEIR BREAKDOWN STRENGTH

By B. M. HOCHBERG and E. J. SANDBERG

(Communicated by A. F. Joffé, Member of the Academy, 15. V. 1946)

The study of breakdown in gases of high dielectric strength is of great interest in solving the problem of the discharge mechanism under pressures close to atmospheric. As is known, the picture of the discharge suggested by Townsend and subsequently developed by Rogovsky and others is not contradicted by experimental data only in the region of low pressures.

Rüdenberg and later Loeb and Meek ⁽¹⁾ evolved a picture of the discharge in air under atmospheric pressure based on the conception of the development of the space charge in a narrow canal of the discharge. The condition for the discharge in this case is the approximate equality of the applied external field and the field created by the space charge in the head of the electron cascade. Only under this condition do the separate little cascades, formed almost simultaneously under the action of photoionization, unite into a general canal, along which the discharge then occurs.

The intensity of the internal field is in the main proportional to $e^{\alpha x}$, where α is Townsend's alpha and x is the length of the cascade, in the limiting case equal to the distance between the electrodes.

Since α in its turn increases very sharply with the growth of the field, the authors considered that equality of the internal and external fields for gases of different dielectric strength is practically attained at values of α differing very little one from another.

Indeed, a variation of α from $\alpha=18$ to $\alpha=20$, i. e. by only 10 per cent, changes the internal field 7.4 times, if the cascade is 1 cm long.

We assumed that this nearly invariable value of the coefficient α , necessary for breakdown, is attained in different gases at different electric fields which determine the breakdown strength of the gases.

To test this supposition the authors performed measurements of the dependence $\alpha/p = f(E/p)$ in the region of fields corresponding to breakdown under normal conditions in air and in a number of gases and vapours of considerable dielectric strength. It must be pointed out that such data exist only for air, nitrogen, oxygen, neon and argon ⁽²⁻⁶⁾, the results obtained in a number of cases being disputable. In gases of considerable breakdown strength such measurements, to the best of our knowledge, have not been carried out.

To measure α the authors chose the usual method of measuring ionization currents. If a current J_0 is created at one electrode owing to initial ionization, the current strength under conditions of a homogeneous field and in the presence of ionization by impact (the distance between the electrodes being x) will be expressed by the well-known formula

$$J = J_0 e^{\alpha x}$$

By performing measurements of the current as depending upon the field intensity for different spark gaps (i. e. $J = f(E)$ for different x), it is possible

to find the dependence of J upon x for different values of the field E , namely, for each value of the field

$$\alpha = \frac{1}{x} \ln \frac{J}{J_0}$$

The main difficulty in these experiments was the creation of a stable high voltage (in our case up to 10 kV) and its regulation within the limits of 10–20 V.

To solve this problem the authors made use of the electric circuit described elsewhere (⁷), with the addition of preliminary voltage control by means of a ferro-resonance stabilizer. The circuit was grounded through a potentiometer, directly coupled to a 300 V battery. This device made it possible to vary the magnitude of the stable voltage by tens of volts, as required for measurements in gases of considerable dielectric strength, for which the whole interval of voltage variation on the curve $J=f(E)$ comprised only

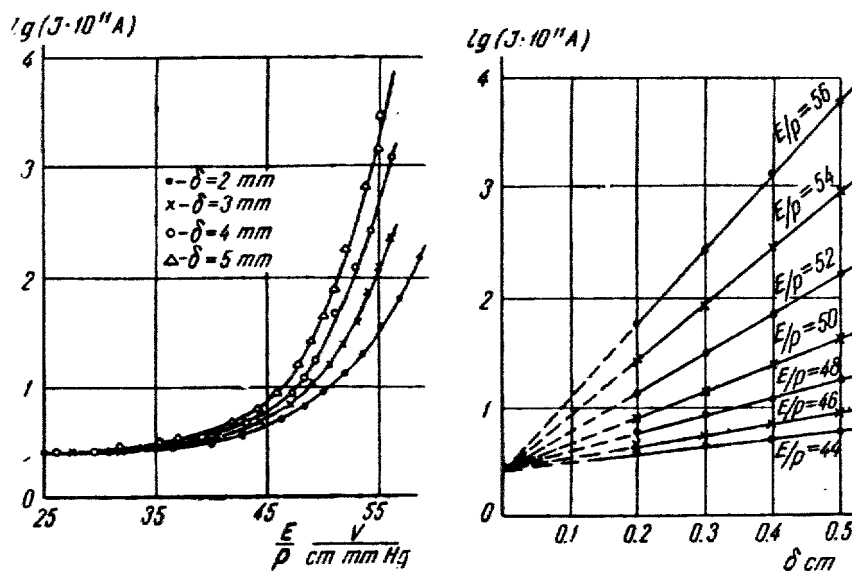


Fig. 1. C_2H_2Cl , $p=300$ mm Hg, $t=18^\circ C$.

200–300 V. Voltage variations during measurements did not exceed 0.1 V at a voltage of 10 kV.

The authors also provided for the exact setting of the electrodes, the creation of a strictly homogeneous field between them and the constancy of the initial current during measurements (in our case, the photocurrent).

The pre-discharge currents were measured by a string electrometer according to Bronson's method.

The choice of substances was restricted by a number of conditions, such as: sufficient vapour pressure, comparative chemical inertness and inconsiderable decomposition during breakdown and under the action of ultraviolet light.

The set-up was tested by measuring the coefficient in air; then measurements were made in SF_6 , C_2H_2Cl , C_2H_2Br , C_2H_2I , $CHCl_3$ and CCl_4 .

Fig. 1 gives a series of typical results of measurements showing the dependences of $J=f(E)$ for C_2H_2Cl when $x=const$ and the recalculated dependences of $J=f(x)$ for $E=const$; these make it possible to calculate α for different E .

Fig. 2 gives the results of the measurements for all the gases investigated by the authors so far. The abscissa gives the values of E/p ; the ordinate, the

values of $\lg(\alpha/p)$. In addition to the data obtained by the authors of the present paper, the broken line gives data for neon, taken from a paper by Glotov^(*)

The straight line parallel to the axis E/p has been drawn for the value of $\alpha/p=0.0215$ corresponding to breakdown in air under normal conditions (field 31.5 kV cm, $E/p=41.5$ V/cm·mm Hg). This value of α/p is attained in other gases under substantially different values of E/p . The authors have calculated the ratio of the fields at this constant value of α/p in all the sub-

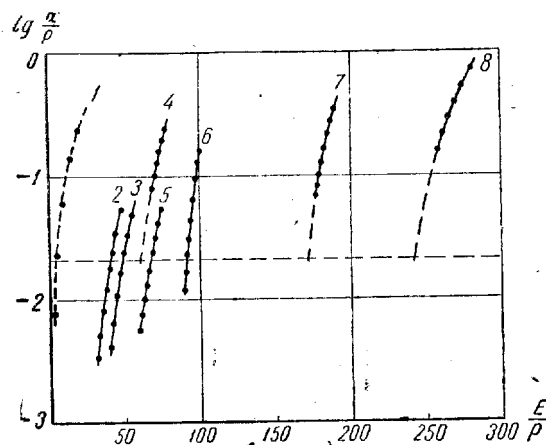


Fig. 2. 1, Ne; 2, air; 3, C_2H_5Cl ; 4, C_2H_5Br ; 5, C_6H_{12} ; 6, SF_6 ; 7, $CHCl_3$; 8, CCl_4 .

stances to the field in air. These values are given in the second column of the table. For comparison the third column of the table gives the values of relative strengths obtained by the authors by direct measurements of the breakdown strength (measurements of the right-hand branches of Paschen's curves and comparisons of the tangents of slopes for rectilinear sections).

Substance	E/E_{air} with respect to fields for $\alpha/p=0.0215$	Relative electrical strength
C_2H_5Cl	1.22	1.23
C_2H_5Br	1.44	1.52
C_6H_{12}	1.63	1.65
SF_6	2.22	2.3
$CHCl_3$	≥ 4.15	4.24
CCl_4	≥ 5.9	6.36
Ne*	0.17	~ 0.14

* According to Glotov's data (*).

A comparison of columns 2 and 3 of the table corroborates the assertion that an express condition for breakdown is the achievement of a definite value by the coefficient α . In gases of considerable dielectric strength this value is attained at correspondingly high fields. In this way the value of the coefficient α chiefly determines breakdown in gases. The authors consider this result extremely important.

In future it is intended to elucidate the conditions of the development of ionization in various gases, since the causes determining the difference in the coefficient α have not yet been explained.

The value of the coefficient α is determined by the rate at which the electrons accumulate the energy sufficient for the ionization of the gas molecules. Since it has been noted that high dielectric strength is typical of the heavy gases with complicated and easily polarized molecules, it can be assumed that the accumulation of energy by the electrons takes place much slower in these gases because of non-elastic losses, much greater electric fields being needed for the beginning of intense ionization.

Leningrad Physical-Technical Institute.
Academy of Sciences of the USSR.

Received
15. V. 1946.

REFERENCES

- ¹ J. M. Meek, Phys. Rev., 57, 722 (1940). ² M. Paavola, Arch. f. Elektr., 22, 443 (1929). ³ K. Masch, ibid., 26, 587 (1932). ⁴ F. H. Sanders, Phys. Rev., 41, 667 (1932). ⁵ D. Q. Posin, Phys. Rev., 50, 650 (1936). ⁶ И. И. Гло-
тов, ЖЭТФ, 7, 1143 (1937). ⁷ Б. М. Гохберг и Э. Я. Зандберг, ЖТФ, 12, 65 (1942).

GEOPHYSICS

**A TENTATIVE INDIRECT DETERMINATION OF THE VELOCITIES
 OF THE TRADE WINDS IN THE EQUATORIAL PART
 OF THE PACIFIC**

By W. B. STOCKMANN

(Communicated by P. P. Shirshov, Member of the Academy, 4. IV. 1946)

In a previous communication⁽¹⁾ the author of the present paper has set forth certain indirect data characterizing the transverse non-uniformity of the trade winds near the equator. These data, obtained on the basis of the theory⁽²⁾ of horizontal circulation in a homogeneous sea, were of a purely qualitative character. It was, therefore, interesting to attempt to calculate the transverse variation of trade wind velocities on the basis of oceanographic measurements. Naturally, such calculations must be based on a more exact theory, taking account not only of the non-homogeneity of the ocean's waters, but also of the effect of lateral turbulent friction, which plays an important rôle in the dynamics of sea currents⁽³⁾.

Considering a steady horizontal movement in a certain region of the non-homogeneous ocean, we may describe such a movement by the following system of differential equations

$$\left. \begin{aligned} A_l \frac{\partial^2 u}{\partial y^2} + \frac{\partial}{\partial z} \left(A_z \frac{\partial u}{\partial z} \right) + 2\rho\omega v \sin \varphi &= \frac{\partial p}{\partial x} \\ A_l \frac{\partial^2 v}{\partial x^2} + \frac{\partial}{\partial z} \left(A_z \frac{\partial v}{\partial z} \right) - 2\rho\omega u \sin \varphi &= \frac{\partial p}{\partial y} \end{aligned} \right\} \quad (1)$$

together with the continuity equation

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \quad (2)$$

In equations (1) and (2) u and v denote the horizontal velocity components in the direction of the axes X and Y of a Cartesian system of coordinates (X pointing eastwards; Y , northwards); p is the pressure; ρ is the density of the water; φ is the latitude of the location, considered constant for a small region; ω is the angular velocity of the Earth's rotation; A_z is the coefficient of turbulent friction due to the exchange of momentum along the vertical Z . The quantity A_z can vary along the vertical according to any law. Finally, A_l designates the coefficient of lateral turbulent friction caused by the exchange of momentum in horizontal directions. Determination of A_l reveals that this quantity undergoes much smaller changes than A_z . Consequently, A_l with sufficient accuracy can be considered a constant quantity.

Differentiating the first equation in (1) with respect to y , the second—with respect to x , and subtracting the second from the first, we obtain by virtue of the continuity equation (2) the expression

$$A_l \left(\frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial x^3} \right) + \frac{\partial}{\partial y} \left[\frac{\partial}{\partial z} \left(A_z \frac{\partial u}{\partial z} \right) \right] - \frac{\partial}{\partial x} \left[\frac{\partial}{\partial z} \left(A_z \frac{\partial v}{\partial z} \right) \right] = 0 \quad (3)$$

in this way eliminating the gradients of pressure and the diverting force of the Earth's rotation.

Let us, now, integrate expression (3) within the limits $z=0$ (the ocean's surface) and $z=H$, the depth at which (and below which) the isobars are horizontal and there is no movement.

Proceeding on the assumption that under the conditions of the ocean we may with sufficient accuracy write

$$\int_0^H \frac{\partial}{\partial y} [F(z, x, y)] dz \cong \frac{\partial}{\partial y} \int_0^H F(z, x, y) dz$$

$$\int_0^H \frac{\partial}{\partial x} [F(z, x, y)] dz \cong \frac{\partial}{\partial x} \int_0^H F(z, x, y) dz$$

(3) can be re-written thus:

$$A_t \int_0^H \left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x^2} \right) dz + \frac{\partial}{\partial y} \int_0^H \frac{\partial}{\partial z} \left(A_t \frac{\partial u}{\partial z} \right) dz - \frac{\partial}{\partial x} \int_0^H \frac{\partial}{\partial z} \left(A_t \frac{\partial v}{\partial z} \right) dz = 0 \quad (4)$$

In the discussion that follows we shall bear in mind that

$$\left. \begin{array}{l} \text{for } z=0 \quad A_t \frac{\partial u}{\partial z} = -T_x, \quad A_t \frac{\partial v}{\partial z} = -T_y \\ \text{for } z=H \quad A_t \frac{\partial u}{\partial z} = 0, \quad A_t \frac{\partial v}{\partial z} = 0 \end{array} \right\} \quad (5)$$

where T_x and T_y are the components of the tangential stress of the wind on the ocean's surface. At a depth $z=H$ the tangential stresses are equal to zero owing to the absence of motion there. As result we obtain from (4) and (5)

$$A_t \int_0^H \left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x^2} \right) dz = -\text{curl } T \quad (6)$$

where

$$\text{curl } T = \frac{\partial T_y}{\partial x} - \frac{\partial T_x}{\partial y} \quad (7)$$

thus likewise eliminating the coefficient of vertical turbulent exchange.

It should be noted that in the equatorial part of the Pacific the zonal components of both the wind and the current prevail. Consequently, assuming $T_y=0$ and $v=0$, we obtain instead of (6)

$$A_t \int_0^H \frac{\partial^2 u}{\partial y^2} dz = \frac{dT_x(y)}{dy}$$

or, with accuracy sufficient for practical purposes,

$$A_t \frac{d^2 S_x}{dy^2} = \frac{dT_x(y)}{dy} \quad (8)$$

where $S_x = \int_0^H u dz$ (elementary flow).

Integrating (8) we obtain

$$T_x(y) = A_t \frac{d^2 S_x}{dy^2} + C \quad (9)$$

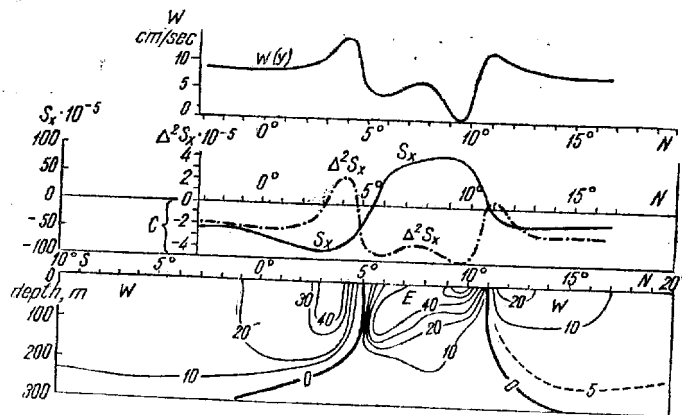
where C is an arbitrary constant.

Let us make use of formula (9) to calculate the transverse non-uniformity of the trade winds in the equatorial part of the Pacific. We shall proceed from the distribution of current velocities calculated by Sverdrup⁽⁴⁾ from observations carried out on board the «Carnegie» along a meridian section made across the equatorial currents (longitude 150° W). Unfortunately, the author of the present paper did not have on hand the tabular values of the current velocities and in his tentative calculation according to formula (9) was compelled to use values of the velocities, carefully reproduced from the isotach graph given by Sverdrup⁽⁴⁾.

This isotach distribution, copied from⁽⁴⁾, is depicted in the lower part of the figure (velocity in cm/sec.).

It goes without saying that our computation, based on a chart so unsuitable for exact calculations, is very approximate.

The middle part of the figure shows the change in elementary flows S_x , determined for a layer extending from the surface to a depth of 400 m, where there is practically no motion. The same part of the figure depicts the change of the second derivative of S_x with latitude (Y-axis), in calculating which we confined ourselves to the second difference of the quantities S_x ($\Delta^2 S_x$). The constant C , which appeared in (9), can be determined graphically by shifting the reference point for $\Delta^2 S_x$ down so that the whole curve $\Delta^2 S_x$ should lie above the abscissa, the absolute minimum of $\Delta^2 S_x$ being taken as the new reference point, corresponding to the zero value of T in the calm belt.



This determined the latitude corresponding to the position of the calm belt.

The latitude of the calm belt on the graph, $\varphi = 9.5^\circ \text{N}$, is in good agreement with its generally accepted position in the Pacific (7–10°N for the summer in the northern hemisphere). We have assumed A_1 to be equal to $7 \cdot 10^7 \text{CGS}$. According to the data of Montgomery and Palmén⁽⁵⁾, this value corresponds to the region of the equatorial counter-current in the Atlantic (there is no determination of A_1 for the equatorial zone of the Pacific Ocean). The author then calculated the velocity of the wind W , making use of the well-known dependence

$$T = 32 \cdot 10^{-7} W^2$$

in which T and W are expressed in the CGS system.

The meridian change in the velocity of the trade winds thus obtained (in m/sec.) is depicted in the upper part of the figure (the wind is perpendicular to the drawing).

As we see, the transverse non-uniformity of the trade winds near the equator is still characterized by the presence of two maxima of the wind velocity

³ C. R. Acad. Sci. URSS, 1946, v. LIII, № 6.

along the sides of the calm belt, maxima predicted and explained in the previous paper (1). The position of the maxima corresponds, as can be seen from the figure, to the latitudes $\varphi_1 \cong 4^\circ \text{N}$ and $\varphi_2 \cong 11^\circ \text{N}$. It is interesting that the southern maximum (14.7 m/sec.) exceeds the northern maximum (12 m/sec.) of the wind velocity.

Our computation has also resulted in the discovery of a new small maximum of the wind ($\sim 6 \text{ m/sec.}$) at a latitude of $\varphi_3 \cong 7.8^\circ \text{N}$ in the region of the equatorial counter-current. At present it appears difficult to explain the origin of this maximum; quite possibly it is a consequence of the squalls, occasionally observed in the calm belt. None the less it must be kept in mind that the field of masses and velocities in the hydrosphere, on the whole, reflects the average state of the atmosphere above the ocean for a long period (of the order of a season). It must also be noted that details of the averaged circulation of the atmosphere can only be obtained on the basis of the statistical treatment of a great number of meteorological observations, not always feasible in the open part of the Pacific.

That is why the indirect method of determining the prevailing non-uniformity of the trade wind velocity, chosen by the author, has certain advantages over direct meteorological observations. Comparing the upper and lower parts of the figure, we see that the equatorial counter-current is mainly (excluding the narrow calm belt) directed against the wind. This circumstance has a definite effect on the distribution of current velocities: the maximum of the velocity of the equatorial counter-current, situated on the surface in the calm belt ($\varphi \cong 0.5^\circ \text{N}$), has appreciably shifted downwards in the southern part of the flow.

Institute of Oceanology,
Academy of Sciences of the USSR.

Received
8. IV. 1946.

REFERENCES

- ¹ W. B. Stockmann, C. R. Acad. Sci. URSS, 53, No. 4 (1946).
- ² W. B. Stockmann, *ibid.*, 52, No. 4 (1946).
- ³ C. G. Rossby, Papers in Physical Oceanogr. and Met., 5, No. 1 (1936).
- ⁴ H. U. Sverdrup, The Oceans, their Physics, Chemistry and General Biology, N. Y., 1942.
- ⁵ R. B. Montgomery and E. Palmén, J. Marine Research, 8 (1940).

TECHNICAL PHYSICS

**UNIDIMENSIONAL SCATTERING OF DETONATION PRODUCTS
 IN THE CASE OF LOCAL ACTION EXPLOSIVES**

By K. P. STANIUKOVICH

(Communicated by P. L. Kapitza, Member of the Academy, 29. IV. 1946)

A study of unidimensional motion of detonation products can be carried out proceeding from the general solutions of the equations of dynamics

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0, \quad \frac{\partial \ln \rho}{\partial t} + u \frac{\partial \ln \rho}{\partial x} + \frac{\partial u}{\partial x} = 0$$

$$\frac{\partial (p \rho^{-n})}{\partial t} + u \frac{\partial (p \rho^{-n})}{\partial x} = 0$$

Here p is pressure; ρ , density; u , the velocity of the detonation products. If the outflow of the detonation products into a vacuum be considered (which is legitimate, since the pressure of the air is small in comparison with the pressure developed on detonation), then we shall have $p \rho^{-n} = \text{const.}$ After the insertion of $c^2 = dp/d\rho$ the above equations transform into

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{2c}{n-1} \frac{\partial c}{\partial x} = 0, \quad \frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - \frac{n-1}{2} c \frac{\partial u}{\partial x} = 0$$

It has been shown earlier⁽¹⁾ that in the case of condensed local action explosives the polytrope exponent n is close to the number 3. With $n=3$ the above equations may be written as follows

$$\frac{\partial (u \pm c)}{\partial t} + (u \pm c) \frac{\partial (u \pm c)}{\partial x} = 0 \quad (1)$$

Denoting $\alpha = u + c$, $\beta = u - c$, we can write the general solutions of equation (1) in the form

$$x = \alpha t + F_1(\alpha) \quad (2)$$

$$x = \beta t + F_2(\beta) \quad (3)$$

The scattering of the detonation products can be represented schematically in a rather simple manner. With an arbitrary position of the detonator inside a cylindrical charge of unit cross-section there arise two waves on detonation, one of which moves to the right of the detonator, and the other to the left.

In the first place we shall consider the wave that spreads to the right. This wave is known to be described by Riemann's solution and characterized by the fact that its parameters are functions of x/t .

When the wave reaches the boundary of the charge, the detonation products begin to fly asunder. A feature distinctive of their scattering is that the velocity of the particles rises abruptly to its maximum value, defined, as we know, by Riemann's solution. On the other hand, the density, or the local velocity of sound, which varies directly as the density, falls as abruptly to zero.

The distribution of velocity and density cannot be found unless we use the general solutions of the equations of gas dynamics. This becomes apparent from the following consideration. When the detonation wave reaches the charge boundary, it gives rise to two waves. One of them may be interpreted as a rarefaction wave moving inward from the boundary of the charge, while the other wave spreads out into space. So Riemann's solution can no longer hold good here, since it is only true for a wave moving in one direction with constant parameters at its front, whereas the parameters at the front of the rarefaction wave running into the charge are gradually decreasing. Two conditions have to be imposed upon the general solution. First, the parameters should be distributed along the vertical, and secondly, the new solution must be conjugated with the old solution of Riemann. If the detonation products scatter into a vacuum, the process can be described completely by means of the two equations of gas dynamics used heretofore. But if instead of vacuum we have a medium of a given density, a shock wave of variable amplitude will arise before the front of the detonation products. The entropy at the front of this shock wave varies continuously, and a complete solution of the problem can no longer be obtained unless we proceed from three equations of gas dynamics.

For the present we shall consider the case of detonation products flying into a vacuum. The new solution referred to above will hold until the front of the rarefaction wave attains the point of weak discontinuity in Riemann's solution for the detonation wave. When this occurs, a new solution appears. This one is again a Riemann solution, since beyond the point of weak discontinuity the pressure and, accordingly, the local velocity of sound remain constant and the velocity of particles is identically zero.

Therefore, the wave moving into the charge will have constant parameters at its front, or, in other words, enjoy properties characterizing a Riemann wave. This Riemann solution gives the relation for the particles' velocity and the local velocity of sound not as a function of x/t , but in a more complicated manner.

Similar reasoning can be applied to the left end of the charge. As a result, we shall find that two Riemann waves of rarefaction—one moving from the right end to the left and the other from the left end to the right—will eventually meet together. At the instant of encounter a new solution arises which is Riemann's solution again and can only be found from the general integral of the equations of gas dynamics. This integral can be found if we proceed from two boundary conditions of its conjunction with the right and left Riemann waves.

As a result, we shall have five equations conjugated with one another at four points. It will later become apparent that the extreme and middle solutions will in time extend over intervals which increase directly as the time, whereas the middle Riemann solution will continue in existence on intervals which retain constant and finite solutions.

Therefore, as t tends to infinity, the masses involved in these integrals tend to zero, and may be neglected in the further discussion.

Thus, a complete solution consists of seven distinct solutions.

Let the detonation begin at a plane passing through the origin. Denote the right end of the explosive by a , and its left end, by b . Then the detonation wave moving from left to right may be described by the equations

$$\begin{aligned} \alpha = x/t, \quad \beta = -D/2 \quad \text{if} \quad D \geq x/t \geq D/2 \\ \alpha = 0, \quad \beta = -D/2 \quad \text{if} \quad x/t \leq D/2 \end{aligned}$$

At the instant $t_1 = a/D$ the detonation wave will reach the right end of the charge, whereupon the detonation products will begin flying

asunder. Proceeding from the general solutions one may state that the process of their scattering is characterized by the equations

$$\alpha_1 = x / t \quad (4)$$

$$\beta_1 = D \frac{x-a}{Dt-a} \quad (5)$$

The first of these equations is obvious, and the second can be derived from equation (3). In fact, for $t_1 = a/D$ and $x_1 = a$ we have $F(\beta) = a - \beta \frac{a}{D}$, and this relation for $t \gg a/D$ gives equation (5).

At the instant $t_2 = \frac{3}{2} \frac{a}{D}$ for $x_2 = \frac{3}{4} a$ the rarefaction wave arising on scattering and moving from right to left will meet the point of weak discontinuity in the solution written for α . This gives rise to a new solution $\beta_2 = \beta_1$ (as before). The new solution obviously is

$$\alpha_2 = D / 2 \quad (6)$$

A similar picture will be observed at the left end of the charge. To write down the corresponding equations we have only to replace a by b , α by $-\beta$ and β by $-\alpha$.

At the instant $t_3 = \frac{3}{2} \frac{a+b}{D}$ the right and left rarefaction waves will meet at the point $x_3 = \frac{3}{4} (a+b)$, thereby giving rise to a new solution again. The new solution clearly will be characterized by the equations

$$\alpha_3 = \frac{x+b}{Dt-b}, \quad \beta_3 = \frac{x-a}{Dt-a} \quad (7)$$

Let us now write all solutions in the old variables u and c and fix the limits of their existence for $t \gg t_3$ (for the right end of the charge)

$$\begin{aligned} u_1 &= \frac{x}{2t} + \frac{D}{2} \frac{x-a}{Dt-a}, & c_1 &= \frac{x}{2t} - \frac{D}{2} \frac{x-a}{Dt-a}, & Dt \gg x \gg \frac{Dt}{2} \\ u_2 &= \frac{D}{2} \left[1 + \frac{x-a}{Dt-a} \right], & c_2 &= \frac{D}{2} \left[1 - \frac{x-a}{Dt-a} \right], & \frac{Dt}{2} \gg x \gg \frac{Dt}{2} - \frac{3b}{2} \\ u_3 &= \frac{D}{2} \left[\frac{x+b}{Dt-b} + \frac{x-a}{Dt-a} \right], & c_3 &= \frac{D}{2} \left[\frac{x+b}{Dt-b} - \frac{x-a}{Dt-a} \right] & \frac{Dt}{2} - \frac{3b}{2} \gg x \gg \frac{Dt}{2} + \frac{3a}{2} \end{aligned}$$

At the left end of the charge the picture will be similar.

Let us consider the distribution of energy, impulse and mass of the detonation products flying in opposite directions at a sufficiently large t ($t \rightarrow \infty$). For this purpose we shall examine the expressions

$$J_\alpha = \int_{-a}^{+a} c_1 u_1^2 dx + \int_0^{+a} c_2 u_2^2 dx \quad (8)$$

For $\alpha=0$, $J_0 \sim$ scattering mass (M); for $\alpha=1$, $J_1 \sim$ impulse (I); for $\alpha=2$, $J_2 \sim$ energy (E). After the evaluation of the corresponding integrals for the left end we, finally, obtain

$$M_1 = \frac{\rho_0}{9} (4a + 5b), \quad M_2 = \frac{\rho_0}{9} (5a + 4b), \quad J_1 = J_2 = \frac{4\rho_0 D}{27} (a + b)$$

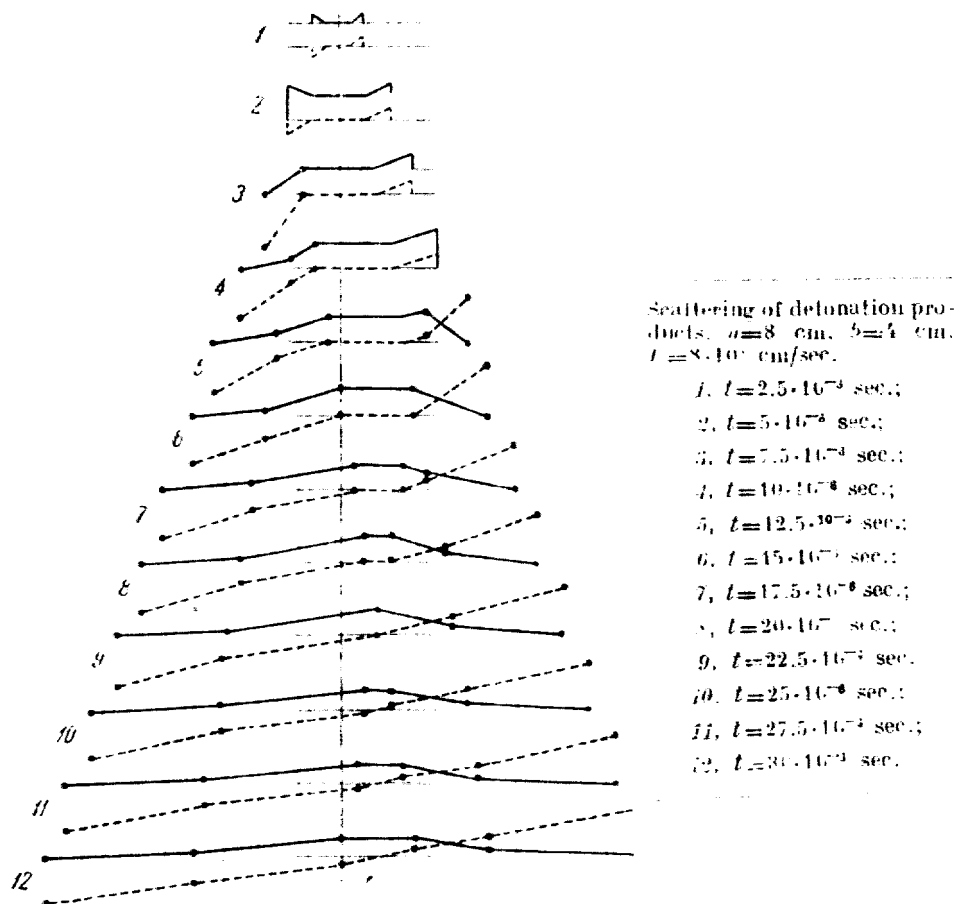
$$E_1 = \frac{\rho_0 D^2}{16 \cdot 27} (16a + 11b), \quad E_2 = \frac{\rho_0 D^2}{16 \cdot 27} (11a + 16b).$$

The equality of the impulses is obvious, since only internal forces are active in the process of detonation. The relations of masses and energies are

$$\frac{M_1}{M_2} = \frac{4a + 5b}{5a + 4b}, \quad \frac{E_1}{E_2} = \frac{16a + 11b}{11a + 16b}$$

For $b = 0$ $\frac{M_1}{M_2} = \frac{4}{5}$, $\frac{E_1}{E_2} = \frac{16}{11}$.

It will be apparent that with the detonator in the extreme position the mass moving toward the charge is smaller than that moving in the opposite direction, but the former carries a greater energy. Thus, a redistribution



of energy takes place in the process of detonation and this distribution is controlled by the position of the detonator. This result is in good agreement with experimental data (2). The distribution of density and velocity of detonation products for different t is shown in the adjoined figure.

Received
29. IV. 1946.

REFERENCES

1. L. D. Landau and K. P. Stanenkovich, C. R. Acad. Sci. URSS, 16, No. 9 (1945).
2. P. H. Hockensmith, Напряжения в детонационных взрывах, 1942.

ELECTRICAL ENGINEERING

**A DEVICE FOR THE SOLUTION OF ORDINARY
DIFFERENTIAL EQUATIONS**

By I. S. BRUK, Corresponding Member of the Academy

The most perfect of instruments known up to date for the solution of ordinary differential equations are based on kinematic principles ⁽¹⁾. Machines of this type exist in many countries ⁽²⁾; they have Thomson's integrating mechanism ^(3,4) as basic link.

Machines with a mechanical integrating element have a high accuracy and are suitable for the solution of a wide class of problems that may be reduced to linear or non-linear ordinary differential equations.

In cases where high accuracy of results is not a necessary condition, an instrument may be used based also on the same kinematic principle but having a more simple integrating element. The reader is supposed to be acquainted with the design and kinematic scheme of the machine for the solution of ordinary differential equations, having a mechanical integrating element ⁽²⁾.

It should be remembered that in this device values of quantities that are related by the differential equation are determined by the angle of rotation of corresponding shafts.

In the instrument described below the integrating element is a scheme with thermionic valves allowing, as in Thomson's integrator, to evaluate integrals of the Stieltjes type $\int y d\psi$, where ψ is a certain function of time. Values of all the quantities involved in the differential equation to be solved are represented by values of an electrical quantity.

To every variable is allocated a certain «bus-bar», the voltage of which with respect to a certain neutral «bus-bar» represents the value of the quantity considered. The buses are connected between them in principle in the same way as the shafts of a machine with a mechanical integrator, and form a «kinematic» scheme in which electric voltages of individual links determine values of variables at any time.

To «connect» the buses elements must be used that can perform summation, integration and multiplication. Functional relations must also be provided for to introduce variable coefficients of the equations. This last operation in mechanical instruments is made by the operator with the aid of a so-called operator panel ⁽²⁾. In electrical devices where the time required by the solution cycle is exceptionally short, as will be shown later on, it is impossible to introduce the coefficients «by hand». Therefore, there is practically only one possibility left (often made use of in the mechanical instrument)—to solve the auxiliary differential equations simultaneously with the main ones, the solution resulting in variable coefficients which are introduced into the solution scheme of the main equation.

It is convenient to begin the description of the instrument design with the integrating element. This is in principle the well-known RC-circuit (Fig. 1). If RC is selected large enough, then for $t \ll RC$

the condenser voltage is $U_c \approx \int e dt$, where e is the voltage on the circuit terminals.

To compute $\int y d\psi$ it is necessary to make $e = y d\psi / dt$. This purpose is served by the balance modulator. If the scheme shown in Fig. 2 is hooked up, the voltage of the condenser C will be equal to $\int y d\psi$. This voltage is fed to the amplifier and a voltage is obtained at the outlet, connected to a resistor load, that may be used in accordance with the requirements of the problem to be solved.

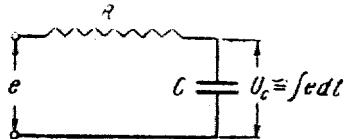


Fig. 1.

A certain time scale must be selected to satisfy the condition $t \ll T = RC$. To make clear how this can be done in the electric instrument, let us refer to the mechanical integrator. In this last the speed of the solution (in time) is known to be determined by the speed of rotation of the independent variable shaft. It is not important whether this shaft rotates at a constant or at a variable speed. Having set the machine going and having reached a certain value of the independent variable, the independent variable shaft may be stopped and by rotating it in the opposite direction the original position may be returned to,

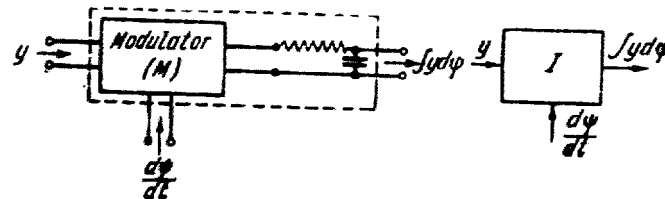


Fig. 2.

for which the initial conditions have been given. After changing the sign of all the initial values of the functions and derivatives and again setting the independent variable shaft in motion, the same solution is obtained, but of opposite sign. The same cycle may be realized with the electrical scheme. This may be illustrated by a comparatively simple example.

The solution is required of the differential equation $dy/d\psi + y = 0$ with initial conditions $\psi = 0$, $y = y_0$. This equation may be re-written in the form

$$y + \int y d\psi = y_0$$

The scheme of solution should have the form shown in Fig. 3. It follows from what has been said above about the mechanical instrument that $\psi(t)$ is a periodic and non-negative function of time. There is a wide scope for the selection of the form of $\psi(t)$. It is convenient to choose a relation that may be easily reproduced. For example: $\psi = \psi_0(1 - \cos \omega t)$.

In this case $d\psi/dt$ will represent a sine voltage curve with a phase of 90° with respect to ψ . The voltage y_0 representing initial conditions must be produced by a rectangular impulse generator at $\omega/2$ frequency (Fig. 4).

If the bus-bar of the independent variable ψ be connected to an a. c. source, and the voltage y be applied to one pair of plates of a cathode-ray oscilloscope and the voltage ψ to the other, the stationary image obtained on the screen will be the (graphic) solution $y(\psi)$ (Fig. 3). Without dwelling on the details

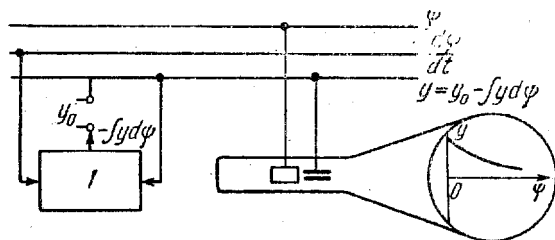


Fig. 3.

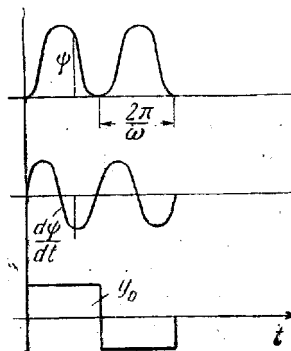


Fig. 4.

of the electric circuit to be realized, the basic problem of time scale will now be reverted to.

To satisfy the condition $t \ll RC$ it is necessary to have variation frequency of ψ sufficiently high (or to have $2\pi/\omega \ll T$, see Fig. 4).

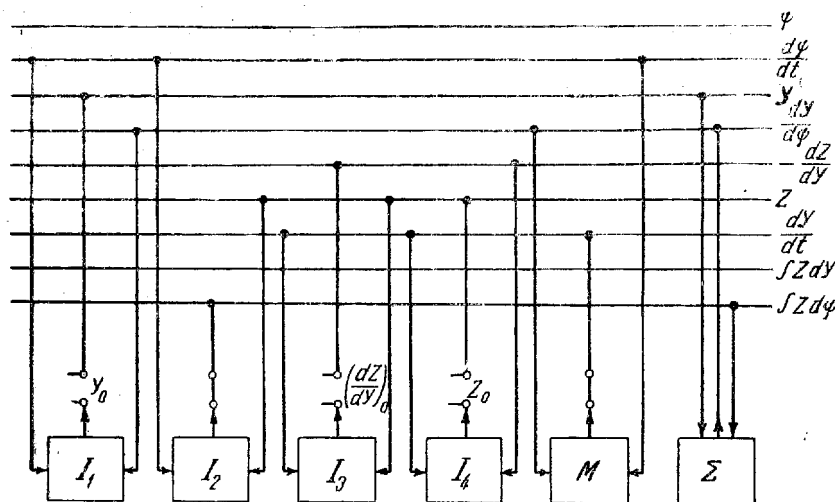


Fig. 5.

Increasing the amplitude ψ_0 , a solution may be obtained for a sufficiently great value of the independent variable without increasing the error due to the integrating circuit.

Consider now a more complex example—a non-linear differential equation

$$\frac{d^2 y}{d\psi^2} + \frac{dy}{d\psi} + \sin y = 0$$

with the initial conditions $\psi = 0, y = y_0, dy/d\psi = y'_0$.

The equation may be re-written as follows:

$$\frac{dy}{d\psi} + y + \int \sin y d\psi = \text{const}$$

The scheme of solution is shown in Fig. 5. $\sin y$ may be determined by solving (simultaneously with the main equation) the auxiliary diffe-

rential equation $\frac{dz}{dy} + z = 0$ with initial conditions $y = 0, z = 0, dz/dy = 1$.

The auxiliary equation in the form

$$\frac{dz}{dy} + \int z dy = 1$$

is introduced into the solution scheme (Fig. 5).

Two integrating elements (I_2 and I_4) and the modulator M serve this purpose; dy/dt is determined by means of the latter.

The summator Σ serves to introduce $dy/d\psi$ into the integrating element I_1 and modulator M . Details of the summator scheme are here omitted.

Only a brief description of the instrument scheme has been given, and its applicability to the solution of some equations shown.

The practical possibilities of application of the instrument are as great, as those of the well-known machine for the solution of differential equations. A detailed description will be published when the experimental work now in hand has been completed.

Received
26. VI. 1945.

REFERENCES

1. И. С. Б р у к, Электричество, № 11 (1945). 2. И. С. Б р у к, Машина для решения дифференциальных уравнений, изд. АН СССР, 1941. 3. William Thomas, Proc. Roy. Soc., 24 (1876). 4. V. Bush and S. H. Caldwell, J. Franklin Institute, 240, No. 4 (1945).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

CHEMISTRY

CONDENSATION OF DIENE HYDROCARBONS WITH BUTYNONE

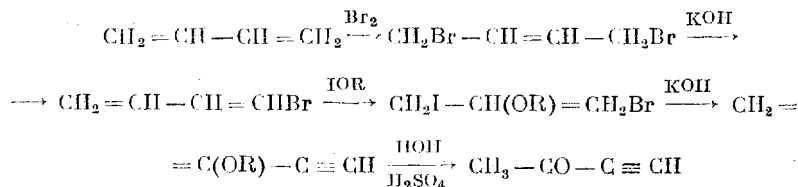
By A. A. PETROV

(Communicated by A. E. Arbuzov, Member of the Academy, 14. III. 1946)

In the author's earlier papers dealing with diene syntheses the reaction of α, β -ethylenic ketones with diene hydrocarbons has been described for the first time. This reaction leads to the formation of hydroaromatic ketones—tetrahydroacetophenone, its homologs and analogs (1).

Further investigations have demonstrated that α, β -unsaturated acetylenic ketones are also capable of condensation with diene hydrocarbons.

The object of our study were butynone, bivinyl, 2,3-dimethyl-butadiene-1.3 and hexadiene-2.4. Butynone was prepared by a method worked out previously starting from bivinyl according to the following scheme (2):



The condensation was carried out in sealed glass tubes at 125—130° in a benzene solution during 12—16 hours. As a result of the reaction only dihydroacetophenone and its homologs have been obtained, the yield being 80 per cent.

Thus, it was proved that butynone and diene hydrocarbons enter the reaction under the conditions specified above, the ratio being only 1:1.




Dihydroacetophenone and its homologs are colourless, strongly refractive liquids, with a peculiar odour much less agreeable than that of tetrahydroacetophenone derivatives. The physical properties of the substances obtained are tabulated below.

It may be seen from the table that the above substances, which are all possessed of a system of conjugated double bonds ($\text{C}=\text{C}-\text{C}=\text{O}$), show an exaltation of the molecular refraction.

In the table are also given the melting points of semi-carbazones, *p*-nitrophenylhydrazones and 2,4-dinitrophenylhydrazones of all the ketones obtained. The velocity of the formation of all these derivatives depends considerably on the position of substituting groups: 2,5-dimethyl- $\Delta^{3,6}$ -dihydroacetophenone reacts much more slowly than 3,4-dimethyl- $\Delta^{3,6}$ -dihydroacetophenone.

One of the interesting particularities of dihydrophenones is their capacity of easy dehydrogenation when distilled over sulphur with the evolution of hydrogen sulphide and the formation of fatty-aromatic ketones. Thus, from dehydroacetophenone acetophenone was obtained (b. p. 198—

—200°, $d_4^{20}=1.029$, $n_D^{20}=1.5340$, m. p. of the *p*-nitrophenylhydrazone 184.5—185°, these data being in agreement with data found in literature); 3,4-dimethyl-2,3,4-dihydroacetophenone yielded 3,4-dimethylacetophenone, b. p.

Formula	B. p. at 20 mm. °C.	d_4^{20}	n_D^{20}	MR		M. p., °C.		
				calculat- ed	found	semicar- bazone	<i>p</i> -nitro- phenyl- hydrazo- ne	dinitro- phenyl- hydrazo- ne
 <chem>CC(=O)c1ccccc1</chem>	95–95.5	1.0126	1.5110	36.02	36.12	199.5– 200.5	203–204	10
 <chem>CC(=O)c1cccc(C)c1</chem>	102–103	0.9681	1.5013	45.25	45.17	176–177	184–186	151–153
 <chem>CC(=O)c1ccc(C)cc1</chem>	127.5–128	0.9868	1.5118	45.26	45–63	225–226	214–215	234–235

247–250°, $d_4^{15}=1.008$, $n_D^{15}=1.5415$, m. p. of the oxime 85.8–86.4°, m. p. of the semicarbazone 230–231° (data from literature: b. p. 250°, $d_4^{15}=1.008$, $n_D^{15}=1.5413$, m. p. of the oxime 84.5–85°, m. p. of the semicarbazone 233–234°⁽³⁾).

This transformation of dihydroacetophenones agrees well with their supposed structure.

Received
15. III. 1946.

REFERENCES

- ¹ A. A. Петров, ЖОХ, 11, 309 (1941); Н. А. Нащиская и А. А. Петров, ЖОХ, 11, 665 (1941). ² A. A. Петров, ЖОХ, 10, 1682 (1940). ³ Beilstein's Handb. d. Organ. Chem., 1, 323, 472 (1918).

GEOCHEMISTRY

**VARIATION IN DEUTERIUM CONCENTRATION IN THE PROCESS
OF MELTING OF ICE**

By R. V. TEIS

(Communicated by V. G. Chlopin, Member of the Academy, 30. III. 1946)

In some previous papers (¹⁻³) it has already been shown that in natural ice the deuterium concentration decreases and the heavy oxygen concentration increases. The experiments on freezing water under laboratory conditions carried out by the author with Florensky (⁴) showed the possibility of discovering the same phenomenon in the laboratory under definite conditions of freezing (freezing out a small part of water and obtaining ice crystals with a dissected surface). A further study of this phenomenon revealed that such an isotope composition of ice was the result of the freezing of water containing dissolved air, while with a preliminary removal of air from the water freezing brings about an accumulation of both deuterium and heavy oxygen in the solid phase. It was also shown that in natural ice, in a measure subject to melting, an increase in the concentration of deuterium and a certain decrease in the concentration of the isotope O¹⁸ are observed (^{2,5}).

To check and further investigate this latter phenomenon we used snows which had suffered greater or lesser changes in the process of melting—firns, frozen snow crusts, etc. The samples were taken* in the Moscow and Yaroslavl provinces in the spring of 1940. The samples were cleaned by the method of Emeléus and others (⁶), used in our previous work. The determination was made by the same method as that used in the previous work, i. e. by a combined measurement of density and of the refractive index of the sample with the same accuracy ($d_s \pm 0.2 \gamma$; $\Delta n \pm 0.5 \cdot 10^{-8}$; separate determination of about 1.0 γ). The results are given in Table 1.

Out of the seven samples here described, four (Nos. 1, 2, 4 and 7) have an isotope composition, differing from the composition of fresh snow in the same sense which has previously been noted, particularly in the increase in their deuterium content (which in fresh snows is found to be negative). The composition of firn No. 3 closely approaches that of fresh snow, which is accounted for by the low rate of melting, as indicated in the table; this sample gives the composition of the snow at the very initial stage of melting. The same may be true of the remains of snow-drift No. 5 (the sample was taken after the first warm day). It will be more difficult to explain the more than once checked relation between samples Nos. 5 and 6: the composition of the pool found beside the snow-drift suggests that it is not related with sample No. 5, as it yields a picture characteristic of the last stages of melting.

Since we are aware (³) of that the process of freezing in nature is complicated by an accessory process, leading to a considerable deviation in the isotope composition of the ice from the theoretical one, it may be expected that the

* Samples were taken by K. P. Florensky.

change in the composition of ice upon melting is also of a complicated nature and does not coincide with that obtained by theoretical calculation.

In order to clear up this question, laboratory experiments with melting ice were carried out; the ice contained 0.1364 atomic per cent deuterium. While studying the waters of glaciers, in order to explain the increases in

Table 1

No. of sample	Date of sampling	Specimen	Description of specimen	Increase in density, %		
				Δd_s	Δd_D	$\Delta d_{1,18}$
1	12.IV.1940	Coarse-grained firn	Crumbling mass of separated, almost quite disjoined grains of transparent ice, 3—5 mm in diameter. Formed at the ultimate stages of melting of snow in the sunshine, under variable temperature conditions	+4.7	+3.7	+4.0
2	12.IV.1940	Firn ice from road	Snow consolidated under pressure in the final stages of thawing, semi-transparent compact ice lying in a crust directly upon the ground	+2.7	+1.4	+6.3
3	30.IV.1940	Coarse-grained firn, Yaroslavl province	Firn remains of snow. Grain size 3—5 mm. In outward appearance similar to firn No. 1, but formed under conditions of slow melting	-2.4	-6.1	+3.7
4	1.V.1940	Frozen-snow crust, Yaroslavl province	Frozen-snow crust on the snow in a wood. At night freely withstands the weight of a man. Taken in the morning, in a slightly thawing condition. Grain size 0.5—1 mm	0	0	0
5	3.V.1940	Remains of a snow-drift, Yaroslavl province	Granular, crumbling snow, medium stage between a frozen-snow crust and a true firn. Evening after the first warm day	-2.2	-6.4	+4.2
6	3.V.1940	Snow pool beside remains of snow-drift No. 5	Small pool in the afternoon of the first warm day	-6.1	+2.0	-8.1
7	14.V.1940	Firn ice	Compact dull ice from under the last remains of a snow-drift thawing in a shaded spot	+2.1	+0.3	-4.8

density, Eucken and Schäfer (?) calculated the variation in density which must take place with various ratios of the initial and residual amount of ice, and found that the order of these variations corresponded to the increase in density as found in natural glacier waters. The equation

$$\frac{m_2}{m_{a1}} + \frac{m_{a1}}{m_{a2}} \left(\frac{m_1}{m_{a2}} \right)^c = \frac{M}{M_a} \left(1 + \frac{m_{a1}}{m_{a2}} \right)$$

where m_2 is the mass of HDO at any given moment; m_{a1} and m_{a2} are the initial

Approved For Release 2002/07/29 : CIA-RDP80-00926A000200010072-1
amounts of the isotope kinds of water; M and M_a are the residual and the initial amount of ice, respectively; c is a constant equal to

$$[1 + \frac{l_a}{RT^2}(T_a - T)]$$

l , heat of fusion of HDO; T_a , temperature of fusion of HDO has been deduced by Eucken and Schäfer theoretically and has not been checked by laboratory experiments. The existence of a phenomenon complicating the process of freezing may cause a divergence between calculation and experiment, namely, the first portions of the waters which have thawed, may carry deuterium in lower concentrations than those in the original ice, and in the further course of the process of thawing these concentrations may increase at a more rapid rate than it would follow from the calculation.

Several pieces of ice of a similar shape, obtained just before the experiment, with a 0.1364 per cent deuterium content, were placed in a funnel, and the water dropping off was collected according to the fractions. The melting of ice (general volume of water was 117.5 ml) proceeded at a temperature of 17.5°, and continued for 3.5 hours. Separate fractions were cleaned by the same method as the waters in Table 1, and measurements were made on the interferometer. The data of these measurements, compared to the calculation according to the equation of Eucken and Schäfer, are presented in Table 2. A verification of the possible influence of the isotope O^{18} on the figures obtained has shown that its maximum effect does not exceed 2 γ (overevaluating the value obtained); thus, with a separate determination of the part of deuterium and of the isotope O^{18} , instead of the last figure of the deuterium content in the solid phase=18.5 γ as given in the table, a value of 17.0 γ is obtained.

Table 2

Portion of ice	Volume of portion of ice, ml	$\frac{v}{V}$	Change of concentration of D with respect to the initial sample**					
			According to calculation (solid phase)		Found in solid phase		Found in water resulting from melting of ice	
			%	γ	%	γ	%	γ
I	27	—	—	—	—	—	—0.0287	—25.7
II	64	0.77	+0.0008	+0.7	+0.0098	+ 8.7	+0.0082	+ 7.3
III	45	0.22	+0.0024	+2.1	+0.0109	+ 9.7	+0.0099	+ 8.8
IV	11.5	0.099	+0.0076	+6.8	+0.0108	+18.5	—	—

* Here the ratio of the remaining amount of ice to the initial one is denoted in the same way as in (3,4). This denotation corresponds to the M/M_a ratio as given by Eucken and Schäfer.

** The value of the original sample calculated from those found for separate portions of ice is 0.1375 per cent. The figures in the last column are taken with respect to the value 0.1364 per cent found experimentally.

In nature, the process of thawing may take place under various conditions. (1) Thawing may occur, as it takes place in the experiment given in Table 2, with a constant flowing off water. Such is, for instance, the melting of icicles, and in some cases, the thawing of small glacier masses. (2) Still more frequently the melting piece of ice is in permanent contact and equilibrium with the water formed upon melting. Such is, *e. g.*, the melting of river ice and in general the thawing of the ice of various water basins.

An experiment was carried out also with ice melting under these latter conditions. The ice, containing 0.1130 per cent D, was melting in a vessel,

in which also the water formed remained. The temperature of the experiment was 17.5°; the thawing of ice (total volume equal to 134.5 ml) proceeded for 3.25 hours.

Tables 2 and 3 show a total lack of coincidence between the experimental and calculated data. In the first portions of the water formed from the melting of ice the deuterium concentration is considerably lower than in the

Table 3

Portion of ice	Volume of portion of ice, ml	$\frac{r}{V}$	Change of concentration of D with respect to the initial sample					
			According to calculation (solid phase)		Found in solid phase		Found in water resulting from melting of ice	
			%	γ	%	γ	%	γ
I	30	—	—	—	—	—	-0.0217	-19.2
II	60	0.776	+0.0005	+0.4	+0.0020	+1.8	+0.0018	+1.6
III	28	0.331	+0.0031	+2.7	+0.0115	+10.2	+0.0105	+9.3
IV	16.5	0.123	+0.0058	+5.4	+0.0125	+12.6	—	—

original sample. Further on, the increase in concentration proceeds at a considerably higher rate than it is required by calculation. However, if in Table 2 the figures obtained be referred not to the original ice but to the concentration obtained after the removal of the first portion of the liquid, the value found for the last portion of ice will be found to be much nearer the calculated one (9.8 γ instead of 6.8 γ , which approaches much nearer the value calculated, especially if the experimental error of +2 γ be considered). In this way, the decrease in the deuterium content is as it were partly «removed» upon the removal of the first quarter of the total water.

The case is different in Table 3; here in the second stage of melting too the decrease is still felt strongly, although the increment of concentration of D in this case also exceeds the calculated one.

In this way the composition of the waters resulting from the melting of ice, as far as their deuterium content is concerned, is found to be different, according to the stage of melting (strongly diminished in the first stages and increased in the last stages); the lower deuterium content in the first stages persists for a longer time, if the melting takes place in contact with the water due to the melting. The increase in the concentration of deuterium in the solid phase proceeds at a much more rapid rate than it could be expected from the ratio of the initial and residual amount of ice.

The W. I. Vernadsky Laboratory of Geochemical Problems,
Academy of Sciences of the USSR.

Received
30. III. 1946.

REFERENCES

- ¹ A. H. Бродский, О. К. Сларре, Е. И. Донцова и М. М. Слункин, ЖФХ, 16, 731 (1937). ² R. V. Teis and K. P. Florensky, C. R. Acad. Sci. URSS, 28, 70 (1940). ³ С. Г. Демиденко, ЖФХ, 14, 380 (1940). ⁴ R. V. Teis and K. P. Florensky, C. R. Acad. Sci. URSS, 32, 199 (1941). ⁵ А. Н. Бродский, Тезисы докладов к совещанию по изотопам 16—17. IV. 1940 г., стр. 5. АН СССР, Отд. хим. наук. ⁶ H. D. Emeléus, F. W. James, A. King, T. G. Pearson, R. H. Purcell and H. V. A. Briscoe, J. Chem. Soc., 1934, 1207. ⁷ A. Eucken u. K. Schäfer, Nachr. Ges. Wiss. Göttingen, Neue Folge, 1, Nr. 11.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

BIOCHEMISTRY

**DAILY VARIATION IN THE ACCUMULATION OF ASCORBIC
ACID BY LEAVES AND FRUITS OF ACTIVE AND INACTIVE
SPECIES OF WILD ROSES**

By M. A. ROSANOVA

(Communicated by A. A. Richter, Member of the Academy, 15. IV. 1946)

In our study of the daily variation in the accumulation of ascorbic acid by leaves and fruits of active and inactive species of wild roses we have been guided by the following idea: (1) we wished to find out, whether the variation in the above process had a definite daily course;

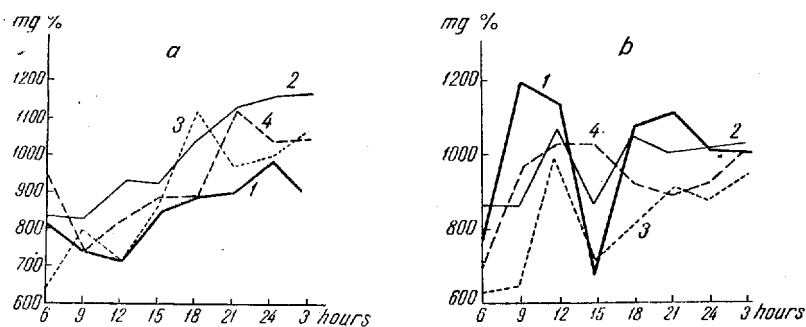


Fig. 1. a, 14.VI; b, 13.VII. 1, *Rosa cinnamomea*; 2, *R. pomifera*;
3, *R. spinosissima*; 4, *R. multiflora*.

(2) whether active and inactive species of roses differed as to their rate and intensity of accumulation of ascorbic acid; and (3) whether it was possible to trace any relation between the daily course of the process in leaves and in fruits.

In the literature we find data on the seasonal variation in the content of ascorbic acid in the leaves and fruits of certain species of roses; but we failed to discover any data bearing upon the daily variation of the same process.

Analyses of the daily content of ascorbic acid were carried out in four species: *Rosa cinnamomea* L. (average content of ascorbic acid per dry weight in the pulp of the fruit, in mg per cent, 7632); *R. pomifera* Herrm. (6449 mg per cent). Among inactive species investigated were *R. spinosissima* L. (1714 mg per cent), and *R. multiflora* Thunb. (657 mg per cent). The content of ascorbic acid was determined by titration of acid extracts with Tilman's

indicator, 0.001 *N* solution of 2,6-dichlorophenolindophenol. The analysis was repeated 8 times daily, beginning with 6 a. m. every 4th hour. A total of four such daily analyses of leaves have been performed within the summer season, and 3 daily analyses of fruits. The analyses were made at the following development stages of the plants: 1. on June 14, budding; 2. July 13, end of blooming and young fruit; 3. August 17, ripening fruit (stages II and III); 4. September 7, almost or entirely ripe fruits (stage III and IV). The stage of ripeness was determined from the colour of the fruits: stage I, young fruits directly after the end of blooming; II, slightly coloured fruits, red fruits in *Rosa spinosissima*; III, orange or orange-red fruits, dark violet in *R. spinosissima*; IV, red fruits, black in *R. spinosissima*. Leaves and fruits from the same shrub were used for analysis.

Curves of Fig. 1 and 2 illustrate the daily variation in the rate and intensity of accumulation of ascorbic acid in leaves; curves of Fig. 3, the course of this process in the fruits. The amount of ascorbic acid accumulated is given

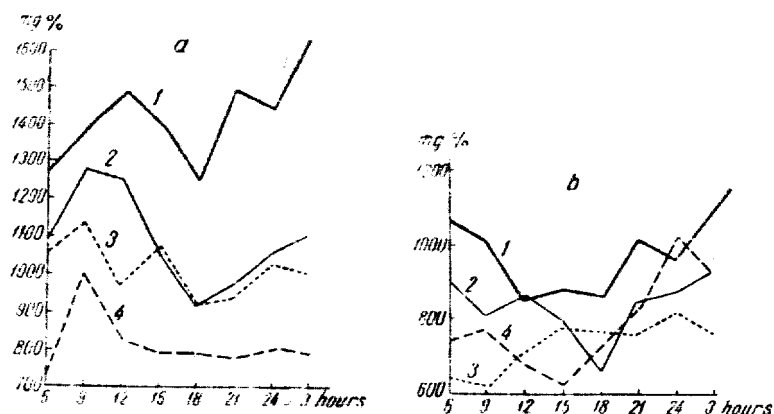


Fig. 2. a. 17. VIII; b. 7. IX. Denotation same as in Fig. 1.

in mg per cent per dry weight of the leaves or pulp, respectively. The data of the analysis of fruits (17.VIII) are omitted, for they differ but slightly from those obtained on September 7.

The examination of the curves illustrating daily variation in the accumulation of ascorbic acid by leaves and fruits permits of the following conclusions:

1. At the phase of budding (14.VI) *Rosa cinnamomea*, which is the species showing the highest content of ascorbic acid in the fruit, has the lowest content of ascorbic acid in the leaves; the highest place is assumed by the curve for *R. pomifera*, which stands next as to its activity. The curves of the least active species, *R. spinosissima* and *R. multiflora*, assume an intermediate position. At the end of flowering and setting of fruits (13.VII) the curve for *R. cinnamomea* is interlacing with those for other species, but in general it runs above them. The curve for the active species *R. pomifera* and that for the inactive *R. multiflora* are close to each other; the lowest place is assumed by that for *R. spinosissima*. At the phase of ripening of fruits of the II and III stage (17.VIII) the curves illustrating the variation in the accumulation of ascorbic acid in leaves are arranged in the order of the activity of the respective species as to the accumulation of ascorbic acid in the fruits. The uppermost place is assumed by the curve for *R. cinnamomea*, the lowest, that for *R. multiflora*. At the phase of ripe or nearly ripe fruit of the III and IV stage the curves run close to each other again, and begin to interlace with

each other. *R. cinnamomea* and *R. pomifera* occupy the first places; the curves for *R. spinosissima* and *R. multiflora* pass nearly at the same level.

2. The run of the daily variation in the content of ascorbic acid in the leaves shows that the least content of ascorbic acid in the leaves usually corresponds to day hours, occasionally even a sharp depression can be observed, as for example, on August 13. At morning hours and especially in the evening the content of ascorbic acid shows an increase.

3. The curves illustrating the daily variation in the accumulation of ascorbic acid in leaves do not coincide with those for the accumulation of this

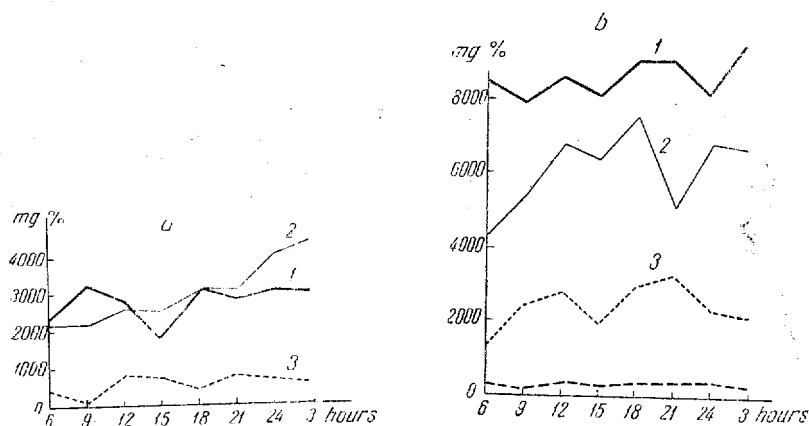


Fig. 3. a, 13. VII; b, 7. IX. Penotation as above.

acid in the fruits. A certain coincidence can be observed at the stage of young fruits only (13.VII). This circumstance may possibly be due to the fact that, in spite of a careful visual selection of the fruits to be analysed, the fruits of the same stage of ripeness may differ in quality, which prevents from establishing the real run of daily variation in the content of ascorbic acid in fruits. According to Zepkova (⁴), the content of ascorbic acid in the fruits of *Rosa* sp. shows an increase towards the noon, but our data do not corroborate this observation.

4. At the stage of green fruits (I stage) the content of ascorbic acid in active species exceeds that of inactive ones. The curves for the active species (*R. cinnamomea* and *R. pomifera*) are interlaced, however, and do not coincide with those for the activity of these species at the stage of ripe fruits.

From what has been said above the following conclusions have to be made:

1. The curves of the daily variation in the accumulation of ascorbic acid in leaves demonstrate a decrease in the amount of ascorbic acid during the day hours, reminding the curves for the daily run of photosynthesis. The only difference consists in that the curves of variation in the content of ascorbic acid tend to rise at evening hours. The latter circumstance is evidently accounted for by the fact that the accumulation of ascorbic acid continues at the expense of the sugars already formed. The existence of a definite relation between the accumulation of ascorbic acid and the process of photosynthesis has already been assumed by several authors (^{1,2,5,6}). Lvov, Gutzevich and Panteleyeva (²) believe this relation to be an indirect one, realized through the intermediation of the sugars formed in the course of photosynthesis. We will not enter here into a discussion whether this relation is a direct or an indirect one, but shall merely state that it is traceable, indeed.

2. Active and inactive species differ in the accumulation of ascorbic acid by their leaves in that the former show a more intense accumulation of ascorbic acid in the leaves at the phase of fruit ripening (stages II and III). At this phase we observe a complete correspondence between the content of ascorbic acid in leaves and in fruits.

3. No correspondence between the daily variation in leaves and fruits could be established, because of the different rate of ripening of fruits on the shrub, and of an insufficient accuracy of the visual evaluation of the degree of ripeness of the fruits.

The All-Union Institute
of Vitaminology.

Received
15. IV. 1946.

REFERENCES

- ¹ В. Благовещенский, Бюлл. экп. биол. и мед., 8, 3 (1937). ² Г. Кузнецова-Зарудная, Сб. витаминов, 1937. ³ С. Львов, Г. Гуцевич, А. Пантелеева, Уч. зап. ЛГУ, 75 (1945). ⁴ G. Zepkova, C. R. Acad. Sci. USSR, 48, No. 9 (1945). ⁵ H. Moldtmann, Planta, 30, 3 (1939). ⁶ T. Sugawara, Jap. J. Bot., 10, 3 (1939).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

BIOCHEMISTRY

**PROTEINASE IN PLANTS AS AFFECTED BY BIOGENIC
STIMULANTS**

By A. G. TOSHCHEVIKOVA

(Communicated by A. A. Richter, Member of the Academy, 20. III. 1946)

The influence of biogenic stimulants tending to enhance the activity of catalase has been observed by Blagoveshchensky and Kologrivova (¹). It appears reasonable to assume that other enzymes may likewise behave in a similar way, and the so more so as this has been conclusively borne out by Chikalo's experiments with the proteinase of germinated cotton-plant seeds (²).

The investigation here described was carried out with purified preparations of the proteinase of germinated seeds of *Phaseolus aureus* Roxb., which had been obtained in two different manners. The seeds were allowed to germinate at a temperature of 20° C until they were 4 days old, after which the lot was separated into two portions. One of these was at once ground with water, with the purpose of extracting the enzyme; another was kept for 10 days at a temperature of 3—7° C, after which it was treated in a similar way. The homogeneous pulp obtained in this way was kept upon addition of toluene during 18 hours at room temperature, thoroughly squeezed out with the aid of a press; the enzyme was then extracted during 3 days at a temperature of 30° from the residue added sodium sulphide or cystein, with a triple amount of a 40 per cent ammonium sulphate solution. After this the extract was filtered off, and the ballast proteins were removed by adding ammonium sulphate up to a 40 per cent saturation. The precipitate was filtered off, and the content of ammonium sulphate in the liquid was brought to a 80 per cent saturation, with a pH equal to 6.4. The precipitates thus obtained were collected on the filter and dried over acetone and ether.

The pulp prepared with the shoots of *Phaseolus aureus* Roxb. thus cooled and treated in this way gave 15 g of the enzyme preparation (from 850 g of seeds), while the portion which had not been subjected to chilling (from 200 g of seeds) gave 4 g of the enzyme. The preparations proved readily soluble both in distilled water and in buffer phosphate, with a pH equal to 5—9. Their isoelectric point corresponded to a pH of 4.7. Both preparations showed a biuretic, xanthoproteinic and mylonic reaction for protein. Their nitrogen content attained 18.32 per cent, being higher than that of preparations from *Asclepias speciosa* Torr, which was found to contain 16.5 per cent (³), or of the commercial preparations of papain (15.5 per cent). Such a high nitrogen content is of no uncommon occurrence in vegetable proteins, however.

The strongest splitting influence upon phaseolin was found to correspond to pH values ranging from 5.8 to 8.0. In agreement with the terminology adopted for proteolytic enzymes of plant origin, the proteinase extracted by the present author from *Ph. aureus* Roxb. should be given the name of «phaseolain», and classed together with papain, bromelin, and yeast proteinase.

In order to compare the effect of the two preparations, as well as their response to the influence of biogenic stimulants, portions of the preparations weighing 0.2 g were solved in 15 ml of phosphate buffer solution, with a pH equal to 6.6, and mixed with 10 ml of a 4 per cent phaseolin solution, prepared from seeds of *Phaseolus aureus* Roxb. In one set each flask was added 5 ml of a solution of biogenic stimulants, prepared with a chilled pulp of shoots of *Ph. aureus*, according to the method described in a note by Blagoveschen-sky and Kologrivova (1). Each control flask was added 5 ml of water. Besides this, each flask was also made to contain 3 ml of toluene.

The flasks containing experimental solutions were transferred to ther-mostats: the one set was kept at 24°, another, at 34° C. In the course of the experiments the temperature of the thermostat fluctuated within a range of -1.0° C. The work of the enzyme was evaluated by determining the nitrogen of free amino-groups by van-Slyke's micromethod. The results are summed up in Tables 1, 2 and 3.

From Table 1 it may be seen that at a temperature of 24° C the effect of the

Table 1
Activity of Phaseolin Prepared with Unchilled Shoots
at 24° C as Affected by Biogenic Stimulants

Period of exposure to enzyme, in days	No stimulant			Stimulant added		
	N-NH ₂ , in mg	Increment in N-NH ₂ , in mg	% to initial	N-NH ₂ , in mg	Increment in N-NH ₂ , in mg	% to initial
0	3.93	—	100	3.93	—	100
1	—	—	—	7.72	3.39	186
2	5.89	0.96	124	9.95	6.02	253
3	5.49	1.56	139	9.21	5.31	235

stimulant is sharply to increase the activity of phaseolin, the cleavage reaching its maximum as soon as after two days. Still more illustrative is the work of the stimulant at 34° C, for this temperature proves to be unfavourable

Table 2
Activity of Phaseolin Prepared with Unchilled Shoots
at 34° C as Affected by Biogenic Stimulants

Period of exposure to enzyme, in days	No stimulant			Stimulant added		
	N-NH ₂ , in mg	Increment in N-NH ₂ , in mg	% to initial	N-NH ₂ , in mg	Increment in N-NH ₂ , in mg	% to initial
0	3.93	—	100	3.93	—	100
1	4.59	0.56	114	9.06	5.13	230
2	4.52	0.59	115	9.59	5.66	244

for the enzyme prepared from unchilled shoots, when tested without any stimulant.

The action of the enzyme at 34° C is weaker than at 24° C, while an addition of the stimulant produces a stabilizing effect, and protein cleavage attains its maximum within no more than one day.

In a pulp prepared with chilled shoots activation of the enzyme takes place during the very process of cooling. The addition of the stimulant fails to influence the enzyme in any way, neither acceleration of protein hydrolysis nor increase in resistance to higher temperatures being recorded.

Similar phenomena were observed by Blagoveshchensky in catalase of chilled shoots of cucumber⁽⁴⁾; these phenomena assume particular importance in the light of the views professed by this writer as to the rôle of temporary chilling of plants in the development of varieties resistant to extreme life conditions.

It is of interest to note that under the influence of an enzyme preparation made with chilled shoots the cleavage of proteins proceeds much slower than

Table 3

Activity of Phaseolain Prepared with Chilled Shoots as Affected by Biogenic Stimulants

Temperature, °C	Period of exposure to enzyme, in days	No stimulant			Stimulant added		
		N—NH ₂ , in mg	Increment in N—NH ₂ , in mg	% to initial	N—NH ₂ , in mg	Increment in N—NH ₂ , in mg	% to initial
24	0	3.93	—	100	3.93	—	100
	1	6.76	2.83	172	7.75	3.82	197
	3	17.43	13.50	445	16.82	12.89	430
34	0	3.93	—	100	3.93	—	100
	1	8.32	4.39	214	8.25	4.32	210
	5	23.69	19.76	628	22.47	18.54	570

in the case an enzyme preparation obtained from unchilled shoots is used. The study of the cause of this phenomenon is now in progress.

The figures tabulated above make it evident that in the case of prolonged chilling of a paste prepared with shoots of *Phaseolus aureus* Roxb. down to 3—7° C, they form some substances tending to enhance the proteolytic activity of the enzyme phaseolain, which is contained in these shoots. An increase in the activity of the enzyme is likewise observed when an enzyme prepared with unchilled shoots is acted upon by a preparation of biogenic stimulants, freed from proteinic substances and heated during 30 minutes in an autoclave at a temperature of 120° C. In the course of this process the resistance of the enzyme to higher temperatures is likewise increased.

Received
20. III. 1946.

REFERENCES

- ¹ A. V. Blagoveshchensky and A. J. Kologrivova, C. R. Acad. Sci. URSS, 48, No. 8 (1945). ² И. И. Чикало, Юбил. сборн. Сп.-Аз. гос. ун-та, 1945. ³ Th. Winnick, A. R. Davis and D. M. Greenberg, J. Gen. Physiol., 23, 275 (1940). ⁴ А. В. Благовещенский, Журн. общ. биол., 6, № 4, 217 (1945).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

AGROCHEMISTRY

**ON THE RÔLE OF PHOSPHORIC ACID IN THE ACCUMULATION
OF ESSENTIAL OILS IN THE LEAVES OF *OCIMUM CANUM***

By M. I. KALINKEVICH

(Communicated by D. N. Prjanschnikov, Member of the Academy, 18. IV. 1946)

Many physiological functions of the cell depend on the presence of phosphoric acid and of its compounds. All important processes of the formation of the molecules of the various enzymes, respiration and fermentation are intimately connected with phosphoric acid. An important part is played by phosphorus and by its compounds in the reactions of carbohydrate exchange in the plant organism (^{1, 2}).

Processes leading to the accumulation in the plants of reduced substances of the type of fats or rubber are also connected with phosphoric acid (^{3, 4}).

This rôle of phosphorus is very important in the case of plants yielding essential oils, whose valuable product consists of reduced substances.

The influence of phosphorus upon the formation of essential oils in the leaves of *Ocimum canum* Sims was found in our experiments of 1938—1939 to differ, according to the conditions of nitrogen supply.

The 1938 experiments were conducted on water cultures. The plants of *Ocimum canum* were grown in modified Hellriegel's mixture under identical conditions till the 5th of September. Different conditions of nitrogen and phosphorus supply were created during the period of mass bloom, from 5th to 25th of September (Table 1).

The plants were harvested on September 25, and their leaves were analysed: the essential oils were determined after Ginsberg (¹); sugars, after Bertrand; and nitrogen, after Kjeldahl (²).

Replication was a four-fold one. Average figures were derived from two parallel determinations referred to air-dry matter.

Table 1

Conditions of nutrition, from 5.IX to 25.IX	Weight of leaves of 100 plants, g	Essential oil		Sugars, %		Nitrogen, %	
		%	cm ³ per 100 plants	Reducing	Saccharose	Non-protein	Protein
2 N 2 P	1751.8	5.15	90.22	3.48	0.44	0.58	2.73
2 N 1/4 P	1589.7	4.17	66.20	5.42	1.02	0.83	3.10
1/2 N 2 P	966.6	2.80	27.08	3.58	1.46	0.56	2.71
1/2 N 1/4 P	1720.9	4.62	79.61	6.79	1.05	0.49	2.84

As may be seen from Table 1, the influence of the amount of phosphoric nutrition at the phase of blooming varies with the surrounding conditions, in our case, with nitrogen nutrition.

The increase in the dose of phosphorus from $\frac{1}{4}$ to the double in the Hellriegel mixture with an increased nitrogen supply (double dose) was followed by an increased content of essential oils, reducing sugars, whereas the content of saccharose, non-protein and protein-nitrogen showed a decrease.

With a reduced dose of nitrogen ($\frac{1}{2}$) the effect of phosphorus upon the accumulation of essential oils and sugars appeared to be an opposite one. An increased supply of phosphorus (2 doses of P_2O_5) with a simultaneous decrease in the dose of nitrogen caused a decrease in the content of essential oils and reducing sugars, and an increased amount of saccharose.

The experiment was repeated in 1939. Differences in phosphoric nutrition were created from August 3 till September 3; before this period all the plants were similarly grown in an enriched mixture containing 3 doses of nitrogen and potassium of Hellriegel's mixture. The plants were harvested during the period of mass blooming (3.IX), i. e. they were physiologically younger at the moment of harvest than in 1938 experiment (Table 2).

The results of this experiment support those of the 1938 studies. With reduced nitrogen supply, phosphorus accelerated the process of ageing.

Table 2

Type of nutrition, from 2.VIII to 3.IX	Weight of leaves of 100 plants, g	Essential oil	
		%	cm ³ per 100 plants
(N) 2 P	1629.0	5.54	99.24
(N) $\frac{1}{2}$ P	1299.7	5.25	68.23

In our experiments this was manifest in the cessation of leaf growth and in the sharp decrease in their essential oil content. This may be taken to account for the fact that the effect of phosphorus varies according to the nitrogen supply in practical application of fertilizers.

It has been established in practical work that phosphorus accelerates the process of plant maturation. This was observed, however, only in case small doses of nitrogen fertilizers were administered. Where great doses of nitrogen were given, additional phosphoric fertilizers may probably result in a slower maturation and in longer growing period.

In order to investigate in more detail the influence of phosphorus upon the process of accumulation of essential oils, upon the carbohydrate- and nitrogen-exchange, experiments were arranged in 1940 on vacuum infiltration of salts of phosphoric acid into the leaves of *Ocimum canum*. For the studies were taken leaves, the first upper pair from the branches of the 2nd order. The leaves were fully developed; the plants were at the stage of the formation of the flower buds or blooming. The leaves were taken from the plants at 9-10 a. m., in sunny weather. Each pair of leaves was divided into two samples, and each leaf split lengthwise into halves; the central vein was discarded. Four samples were thus obtained, weighing from 80 to 100 g each. Two of them were used for the water control, the remaining two being infiltrated with a 0.05 N solution of KH_2PO_4 . The technique was that used by Smirnov and Shcherbakov (*). The duration of the experiment was 25 hours; the results are represented in Table 3 (all the figures refer to absolutely dry matter).

Introduction into the leaves of *Ocimum* of a solution of KH_2PO_4 was accompanied by an accumulation of essential oil, of reducing sugars, slightly reduced saccharose fraction. No influence of phosphorus upon nitrogen exchange could be noticed. Additional experiments on vacuum infiltration of potassium salts into the leaves of *Ocimum canum* have shown that the physiological effect of KH_2PO_4 should be attributed to phosphoric acid.

Conclusions. The physiological effect of phosphoric acid contained in the leaves of *Ocimum canum* in the process of the accumulation of essential oil, of sugars and nitrogenous substances varies as to the conditions of nitrogen nutrition.

1. With an increased amount of nitrogen, phosphorus increases the quan-

Table 3

Date	Infiltration	Essential oil, %	Sugars, %			Nitrogen, %		
			Reducing	Saccharose	Total sugar	Non-protein	Protein	Total
19.VIII	H ₂ O	10.69	2.72	1.06	3.78	0.87	4.78	5.65
	KH ₂ PO ₄	11.06	3.40	0.41	3.81	0.86	4.74	5.60
2.IX	H ₂ O	8.84	8.63	1.65	10.28	0.79	3.72	4.51
	KH ₂ PO ₄	10.38	9.07	1.50	10.57	0.80	3.71	4.51
11.IX	H ₂ O	9.97	5.86	6.78	6.64	0.91	4.59	5.53
	KH ₂ PO ₄	10.85	7.44	0.08	7.52	1.01	4.68	5.69

tity of essential oils, whereas a reduced amount of phosphorus decreases the yield of leaves, and impairs the quality of the latter.

2. With shortage of nitrogen in the nutrient medium, a more liberal phosphorus supply reduces the yield of leaves and the percentage of essential oils contained in them; a reduced dose of phosphorus under such conditions increases the number of leaves and the relative amount of essential oils in them.

3. The accumulation of essential oils in the leaves of *Ocimum* shows a positive correlation with reducing sugars. The presence of an increased amount of reducing sugars in the leaves is accompanied by a greater relative amount of essential oils.

4. Vacuum infiltration of a solution of KH₂PO₄ into the leaves of *Ocimum canum* at the period of blooming in experiments lasting 24 hours increased the proportion of the essential oils and of reducing sugars, with a certain reduction in the saccharose content.

The K. A. Timiriazev
Academy of Agriculture,
Moscow.

Received
18. IV. 1946.

REFERENCES

- ¹ Г. И. Гинзбург, Журн. хим.-фарм. пром-сти, № 8—9 (1932).
- ² А. Р. Кизель, Практическое руководство по биохимии растений, 1934.
- ³ Д. И. Прянишников, Агрохимия, 1940.
- ⁴ W. A. Engelhardt, Bull. Acad. Sci. URSS, sér. Biol., No. 3 (1945).
- ⁵ А. И. Смирнов и А. И. Щербачев, Химизация соц. земледелия, № 6 (1938).
- ⁶ И. В. Мосолов, там же, № 12 (1938).
- ⁷ А. Ф. Раппинер, там же, № 10—11 (1939).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1946. Volume LIII, № 6

GEOLOGY

THE POST-PLEISTOCENE AGE OF THE EAST CARPATHIANS

By V. V. BUTZURA

(Communicated by D. S. Beliankin, Member of the Academy, 3. VI. 1946)

It is usually believed that the East Carpathians had been formed during the Tertiary and that in the Miocene they already had the nature of a high land, which has been since subject to a destructive denudation by exogenetic factors. A study of the river steps (terraces) and their remains in the valley of the Transcarpathian Ukraine, irrigating the Transcarpathian south-western slope of the East Carpathians, leads, however, to different conclusions as to the age and development of these mountains.

One of the most characteristic features of the relief is its relative height. Below are given the principal results of a palaeomorphological investigation of the Transcarpathian region of the East Carpathians, which are concerned with the increase in the relative height, and the regularities underlying the development of the East Carpathians, which follow from these data.

On the south-western slope of the East Carpathians in the basin of the mountain river Tissa (¹), between the river level and the highest flattened parts of the high-mountain arch are developed remains of river steps, shelves, and chiefly their homologues. From a study of these forms of relief in the longitudinal profile of the valley of the Tissa, traces of river stages have been found which may be reduced to the following system serving as a working hypothesis: I, the lowest step referring to the Holocene represented at some places by two rows of terraces; II and III steps and the corresponding shelves, to the Upper Pleistocene; IV step and its shelf, to the Middle Pleistocene; V step and shelf, to the Lower Pleistocene; VI step and its shelf (Guntzian) and VII step with its shelf, to the Pliocene. Higher up, flattened forms of the upland arch, referred to the residual plain which formed during the Pliocene and possibly began to form as early as in the Miocene. It represents the VIII step.

Data on the steps and shelves in the valley of the Tissa River show in the first place that the relative height of the East-Carpathian highlands in the profile between the chief divide in the north-east and the Pannonian depression in the north-west increased not with a gradual rise, but stepwise.

The oldest of the periods of growth of the highland established belonged to that age of the Pliocene when the residual plain of the upland arch had finally formed; the recent plane of the latter is now 300—400 m lower than the highest summits of the recent highlands. Vast flattened stretches of the residual plain, which would be more correctly termed a peneplained shelf, show that the relief of this generation of the landscape was a mild one and was very slightly dissected, and that the upland at that times presented a somewhat raised, nearly flat shelf, gently convex following its transverse axis. The maximum heights of the shelf were along the line of the mountain groups Chernogora—Polonina Rovna, south-west of the present main divide

of the East Carpathians. The valleys of this landscape were vast and had an insignificant gradient, while their slopes usually did not exceed 100 m.

There is no evidence of a mountainous character of this generation of the relief, while the data available are in conflict with such an idea of the landscape. The fact is that under conditions of a mountain character of this generation of the relief, deep valleys should have developed, connecting the central parts of the highland with the fringe of the lowland foothills of the mountain system, the gradient of the valleys decreasing in the course of time from the Pliocene to the Holocene. As a matter of fact, there are no data pointing to the presence of deep valleys in the Pliocene, while the system of valley steps developed after the peneplain shelf had formed is evidence of a progressive increase of the gradient in the course of time from the Pliocene to the Holocene. The hypothesis of the formation of the residual plain of the East Carpathians by a destruction of the upland relief in the sphere of the upper denudation level (as understood by A. Penk) thus becomes invalidated. The data on the valley steps permit of a most natural explanation of its formation by a peneplaining denudation of the relief by running waters, under conditions of low relative heights.

After the peneplanation of the shelf, the valleys were deepened by 200 m, and then the VII valley step was worked out. The relative height of this generation of the landscape increased to 600 m, and consequently did not yet attain the value characterizing the highlands. The rivers of this period of development of the highlands were confined to wide valley lowlands. The geographical distribution of the remnants of the VII valley step indicates that at that time in the landscape predominated (structural) longitudinal valleys of tectonic origin with a SE--NW trend, while small sculptural transverse valleys played a subordinate part. At this period the shelf was transformed into mountains of medium height with an undulating relief, the chief elements of which were structural longitudinal mountain ranges and valleys of a similar type.

Similar conditions existed also at the end of the Guntzian, when the relative height of the upland reached 800 m during the formation of the VI valley step. Owing to an increase in the transverse gradient of the highlands, however, in the Guntzian transverse rivers became more strongly developed, and an intense struggle for basins began between these and longitudinal rivers.

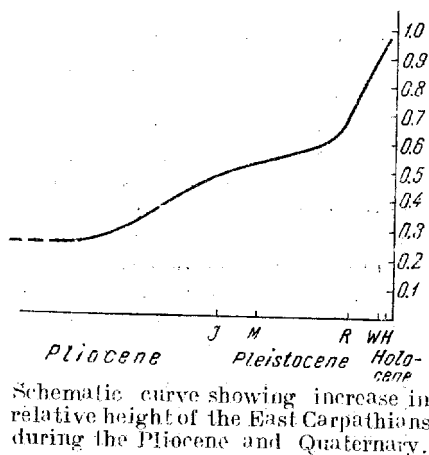
A new uplift caused a deepening of the valleys by 200 m to the level of the V valley step, and the relative height of the highland increased to 1000 m by the end of the Mindelian. The struggle between transverse and longitudinal rivers became very intense. The highlands at its extreme points reached the limiting relative height characteristic of high altitudes. On the average, however, the relative height was less than 100 m, and no deep valleys with steep slopes, typical of high altitudes, had so far been worked out.

It was not until the Rissian age that an altitudinal landscape of the highlands was created after the folded structure of the East Carpathians had been raised again in an abrupt leap, and the relative height of the relief at once reached 1300 m. In this age the displacement of longitudinal rivers by transverse ones was complete: the transverse rivers finished to cut out longitudinal mountain ranges by retrogressive erosion. Having a large gradient, they dissected the ancient longitudinal valleys, transformed into subordinate stretches of the valley network. Thus, in the landscape a predominance of sculptural transverse valleys instead of structural longitudinal ones was established, as a result of which the longitudinal type of dissection of the highlands was replaced by a lattice type.

The Rissian uplift was followed by vigorous Würmian elevations, when the transverse valleys were strongly deepened and the relative height of the highlands increased to 1700 m. In the extreme upper course of the Tissa River it differed but slightly from the present relative height of the highlands,

which has increased there only to 1715 m. Owing to a further development of the network of transverse valleys, some longitudinal ranges were completely destroyed, and the highlands began to assume the type of transverse dissection, the first signs of which were manifest at some places in its marginal portions.

The data on the increase of relative height cited above refer to the indicated profile of the territorially restricted Transcarpathian region of the East Carpathians. The fact that the same process of morphological development was common both to the south-western and the north-eastern slope of the highlands is beyond any doubt, however. This is indicated by the fact that an age correlative relationship is noted even between the system of terraces of the basin of the Tissa River and of the Podolia Dniester (^{3, 4}). The process of the geomorphogenesis common to both slopes of the East Carpathians permits of the conclusion that the development of relative height, the general



features of which have been presented above, is characteristic of the Carpathians as a whole.

This enables us to plot a curve showing the increase in the relative height of the East Carpathians (figure) during the period from the age of the Pliocene when the peneplained shelf was formed, to the Holocene. On the ordinate the present height range of the high mountainous country is put at unity. On the abscissae time is plotted. The curve has been constructed from the average data on the increase in the relative height during the phases of formation of separate shelves without considering the data on the decreases in height, which might have taken place during the phases of formation of the terrace steps.

From the curve in the figure it may be seen that at the period of time characterized three main periods («cycles») of development of the East Carpathians can be distinguished. The first of these terminated at that age of the Pliocene when the East-Carpathian shelf was subject to peneplanation, which transformed it into an almost plain elevation. The second period embraced the time from the age of formation of the most ancient shelf and the VII valley step to the Mindel-Rissian age. The third period characterized the time from the Rissian age to the Holocene, when the East Carpathians were very rapidly rising.

The levelling of the curve during the Holocene probably reflects the subordinate phase of a temporary slackening in the rise of the East Carpathians, analogous to the phases which had been also previously displayed. The short duration of this slackening of elevation might be indicated by the general nature of the curve and chiefly by its very intense intercept between the

Rissian and the Holocene, rising very steeply upward. It would seem, therefore, that we are witnesses of an intense mountain making, which has now somewhat lost in intensity but, judging from the character of the curve, must still be going on.

Subordinate to the three periods of the major order are periods of a minor order (formation of shelves and terraces) with respect both to the duration and the amplitude of elevation of the highlands. The duration of every successive major-order period probably diminishes as compared to the preceding one. Only the second period is reflected on the curve in its full duration. As it will be seen, peculiar to the first phase of this period is the predominance of elevations, and to the second phase (Mindel-Rissian), a relative quiescence, or, more strictly speaking, a certain sinking of the highlands. It may be supposed that an analogue of the second phase of the second period is the phase of peneplanation of the Pliocene shelf at the first period. The third period of development of the East Carpathians is analogous to the first phase of the second period.

The three periods of increase of the highlands are accompanied by three successive, genetically closely related types of dissection of the relief. The longitudinal type of dissection of the Pliocene relief passes through a lattice dissection, peculiar to the present East Carpathians, to a transverse dissection, diametrically in contrast with it. The first signs of the latter became noticeable by the end of the third period of development of the highlands in some areas of its south-western and at places also north-eastern border. They may be the forerunners of a transverse dissection of the East Carpathians in the geological future. The dissection proceeds from the borders towards the central portions of the highlands. At the same time its rate is more rapid on the south-western slope than on the north-eastern one.

From the data adduced above it follows that the types of dissection of the upland relief are developing according to definite laws and have genetic series of their own, one of which is that referred to above (longitudinal—lattice—transverse), and that corresponding to the separate stages in the development of the highlands are definite types of dissection of its relief.

The inference as to the Rissian age of the highlands of the East-Carpathian portion of the Carpathian high-mountain arc, forming one of the mountain systems of the Alpine tectogenesis, is of somewhat wider interest if it be recalled to mind that all the data on the other parts of the Alpine system suggest with increasing conclusiveness that their age is considerably younger than it has previously been believed. It is particularly important to note that the Great Caucasus did not acquire its high-mountain nature until the Rissian and Würmian ages, and that its pre-Mindelien (?) (in the writer's opinion, more ancient) relief had also been a strongly peneplained one.

Received
7. V. 1946.

REFERENCES

- ¹ В. Бунур, *Изв. Ин-та геогр. и картограф.*, 2, 1928/29 (1930). ² Л. А. Варданьянц, *Изв. Рос. геогр. общ.* (1933). ³ Л. Ф. Лунгерстаузен, *Сов. геол.*, № 8 (1940). ⁴ Д. Н. Соболев, *там же*, № 6 (1938).

MINERALOGY

ON CALCAREOUS HEXAALUMINATE FROM ZESTAFONI (GEORGIA)

By D. S. BELIANKIN, Member of the Academy, V. V. LAPIN
and J. P. SIMANOV

The two first authors of the present communication, in collaboration with Shumilo (¹), observed some time ago in the Zestafoni high-alumina slags obtained in smelting chromaluminium highly peculiar crystallization of alumina, which seemed to proceed in the forms of typical corundum, but showed at the same time two rather unexpected features.

1. With a relatively weak birefringence, normal of corundum, the value of N_o varied within a more or less considerable range—from 1.746 to 1.764; it will be seen that only the upper limit corresponds in a measure to common corundum.

2. A chemical analysis of the preparation which consisted almost entirely of the given mineral, with a negligible admixture of the second mineral— β -alumina, having optical constants familiar to us, showed a CaO content up to 5.40 per cent.

At that time we failed to arrive at any satisfactory explanation of this phenomenon. Now that we have resumed the study of the problem and in the light of additional information, more satisfactory results could be obtained.

We have been prompted to this additional study by a paper recently published by Filonenko (²), dealing with the problem of the crystalline phases in a block of Cheliabinsk abrasive electrocorundum. In this block, according to Filonenko, as a result of using in the smelting process low-grade bauxites with 1—2 per cent CaO, along with corundum calcareous hexaaluminate, $\text{CaO} \cdot 6\text{Al}_2\text{O}_3$, was crystallized, laminated as usual (the so-called β -alumina) but with peculiar optic characters: $N_o = 1.757$; $N_e = 1.750$; $N_o - N_e = 0.007$, which are very far from the data on the optic properties of this mineral cited in the literature (³): $N_o = 1.702$; $N_e = 1.667$; $N_o - N_e = 0.035$.

The hexaaluminate of Filonenko has partly corroded and replaced the corundum of the block, in a degree gradually increasing in the direction towards the base of the block. At the bottom of the latter, at its contact with the hearth, it had completely displaced corundum of earlier crystallization, but was accompanied by later corundum of the second generation, which filled the polygonal spaces between the lamellae of the hexaaluminate.

While not satisfying the characters of hexaaluminate as described in the literature, the optic properties of the mineral of Filonenko perfectly fitted within our determinations made on the material from Zestafoni. Our interest being aroused by this fact, we prepared thin-sections which were so far not available from the Zestafoni material. Rather unexpectedly, instead of one, two principal minerals appeared in these thin-sections: (1) typical corundum and (2) hexaaluminate with properties of the mineral of Filonenko. The first of these was in the form of slightly resorbed pegmatite-poikilitic intergrowths in the lamellae of the second (cf. photomicrograph, $\times 46$, in polarized light). As quite negligible accessories, the following were

³ C. R. Acad. Sci. URSS. 1946. v. LIII, № 6.

observed: (1) traces of metal and spinel, and (2) exceedingly thin foliae of a brightly polarizing uniaxial and negative mineral, following the cleavage planes in the hexaaluminate; this mineral from the totality of its properties



Corundum intergrowths in calcareous β -alumina

strongly resembled more common β -alumina. The optic constants of the three uniaxial minerals named above determined by the authors are compared below:

	N_o	N_e	$N_o - N_e$
1. Corundum . . .	1.765	—	—
2. Hexaaluminate . . .	1.742—1.760	—	—
3. β -alumina . . .	1.736	1.748	0.018

Neglecting the accessory admixture escaping precise evaluation, we have calculated in three separate thin-sections the relative volume quantities of corundum and hexaaluminate, which are given below in volume per cent.

	1st thin-section	2nd thin-section	3rd thin-section	Average
Corundum	21.0	18.4	17.3	19.1
Hexaaluminate	79.9	81.6	82.2	80.9

Since, as it will be shown below, the specific gravity of corundum exceeds but very slightly that of hexaaluminate, it does not seem likely that the weight per cent of the two minerals here discussed would differ much from the volume per cent. Approximately, the percentages will be 20 per cent of corundum and 80 per cent of hexaaluminate.

In order to determine the actual quantitative proportions of CaO and Al_2O_3 in the composition of hexaaluminate, we have used a detailed chemical analysis of the slag in question. It was subjected to a preliminary treatment with hydrochloric acid, to remove the metallic admixture from its composi-

tion. The chemical analysis of the preparation thus obtained was then made by V. A. Moleva, with the following results: SiO_2 , traces; TiO_2 , none; Al_2O_3 , 92.85 per cent; Cr_2O_3 , 0.04; Fe_2O_3 , 0.07; MnO , none; MgO , 0.41; CaO , 6.13; Na_2O , traces; K_2O , traces; S , 0.18; loss on ignition, 0.40 per cent.

Referring 20 per cent Al_2O_3 to corundum and 1.05 per cent of the same oxide to spinel (0.41 per cent MgO to each), from the remainder we obtain for hexaaluminate: 71.84 per cent $\text{Al}_2\text{O}_3 + 6.13$ per cent $\text{CaO} = 77.97$ per cent, and for 100 per cent aluminate: 92.14 $\text{Al}_2\text{O}_3 + 7.86$ CaO , instead of the theoretical 91.60 $\text{Al}_2\text{O}_3 + 8.40$ CaO . As may be seen, the result agrees rather well with the microscopical determination and calculations.

The specific gravity of the bimineral slag here discussed (analysed preparation), according to the picnometric determination of Z. G. Khaustova, with the use of alcohol, was found to be 3.92. Since the specific gravity of corundum is 4, to find that of hexaaluminate, X , the following equation was used: $(X \times 80) + (4.0 \times 20) = 3.92 \times 100$, from which $X = 3.90$, a value very considerably differing from the one obtained by Filonenko for her hexaaluminate (3.54).

Because of a considerable dissimilarity between the optic characters of the hexaaluminate of Filonenko and those of the hexaaluminate as described in the literature, she advanced the view that in this particular case «we possibly had a new modification of $\text{CaO} \cdot 6\text{Al}_2\text{O}_3$ or of crystalline alumina». To verify this assumption the third author of the present communication carried out a special X-ray examination of the bimineral slag in question.

After the slag had been finely ground and elutriated, X-ray powdergrams of it were obtained using an ionic tube of the Hadding type, with an iron target, without filter in cameras with approximate diameter of film-holder 57 mm. On the photographs obtained the following crystalline phases were determined.

1. Calcareous hexaaluminate with parameters coinciding with those for β -alumina of Lagerqvist (⁴): $a = 5.536 \text{ \AA}$, $c = 21.825 \text{ \AA}$.

2. Corundum with parameters: $a = 4.752 \pm 0.001 \text{ \AA}$, $c = 13.02 \pm 1.01 \text{ \AA}$. Comparing these values with those cited in the literature: $a = 4.749 \text{ \AA}$, $c = 12.966 \text{ \AA}$ after Jay and Wilde (⁵), as well as with those obtained this time specially for the artificial corundum of Popov (¹): $a = 4.747 \pm 0.001 \text{ \AA}$, $c = 12.963 \pm 0.006 \text{ \AA}$, we find that in the corundum from slag a slight expansion of the lattice has taken place.

In addition to the lines of these two principal phases, in the photographs weak lines $d = 2.143 \text{ \AA}$ and $d = 1.349 \text{ \AA}$ were identified, corresponding to the «strong» lines of alkaline β -alumina of Brownmiller and Bogue (⁶). Other possible lines of it were disguised by considerably stronger lines of the two preceding phases.

Similarly, observed on the photographs was a hardly perceptible line, corresponding to the strongest line 044 of spinel: $a = 807 \text{ \AA}$.

As a result of the microscopic calculations supported by chemical determinations, the above cited weights per cent of corundum and hexaaluminate in the slag were found: 20 per cent of the former, and 80 of the latter. It was very interesting to check up the correctness of these values also by the X-ray method. With this object in view, the third author of the present paper used the method of comparing the intensities of two lines: (116) of corundum and (222) of sodium chloride on the photographs of the following mixtures prepared for this purpose:

I.	60%	Popov's corundum	+ 40%	NaCl	2
II.	48%	»	»	+ 52%	»	1½
III.	82%	»	»	+ 18%	»	1

The figures at the end of each line express the difference in the intensities of the lines, as evaluated by the 10 mark system, every time in favour of corundum. Considering this difference in the first approximation to be pro-

portional to the relative corundum content in the mixture corundum + NaCl, we arrive at the conclusion that in the III mixture, when calculated to 100 per cent corundum + NaCl, 52 per cent corundum + 48 per cent NaCl should be contained, and to the actual 18 per cent NaCl, $\frac{18 \times 52}{48} = 19.5$ per cent corundum. This is for a mixture of 82 per cent slag + 18 per cent NaCl; and for 100 per cent slag, $\frac{19.5 \times 100}{82} = 23.8$ per cent corundum. With a relatively low accuracy of evaluation of the intensities of the lines, *etc.*, the result is found to be in sufficient agreement with the data of microscopy and chemical analysis.

From our brief communication the following conclusions may be made:

1. What we have formerly termed Zestafoni corundum, is actually a mixture of corundum with predominating calcareous hexaaluminate of Filonenko, identical with β -alumina of Lagerqvist.

2. Data cited in the literature on the optical characters of calcareous hexaaluminate, which do not agree with those of the present authors and of Filonenko, have to be revised. The same is true of the specific gravity of hexaaluminate, the value of which as given by Filonenko seems to be too low (cf. in this connexion the history of the investigations of γ -alumina (?)).

3. Still another " β -alumina", present in the Zestafoni slag as an insignificant admixture, alkaline according to X-ray data, has optical characters of its own, not quite similar to those given in the literature. Additional investigations are necessary here too.

4. All this is so much the more desirable, since—as shown by the work of one of us in collaboration with Shumilo (*),—in the molecular composition of aluminates whose properties approximate those of " β -alumina" the proportions observed may be different from $\text{RO} : 6\text{Al}_2\text{O}_3$.

Received
9. VII. 1946.

REFERENCES

1. D. S. Beliankin, V. V. Lapin and I. M. Shumilo, C. R. Acad. Sci. URSS, 30, No. 8 (1941). 2. N. E. Filonenko, *ibid.*, 48, No. 6 (1945). 3. N. A. Turov and M. M. Stukalova, *ibid.*, 27, No. 9 (1940). 4. K. Lagerqvist, S. Wallmark u. A. Westgren, Z. anorg. u. allg. Chem., 234, 1 (1937). 5. A. H. Jay and W. T. Wilde, Min. Mag., 27 (187), 56 (1944). 6. L. T. Brownmiller and R. H. Bogue, J. Res. Bur. of Stand., 8, 289 (1932). 7. D. S. Beliankin, Zbl. f. Min., Abt. A, Nr. 10, 500 (1933). 8. Д. С. Белянкин и И. М. Шумило, Тр. Петр. ин-та АН СССР, 13, 211 (1938).

PETROGRAPHY

ON THE COMPOSITION AND PROPERTIES OF «VAPS»

By N. A. IGNATIEV and A. M. KUZNETZOV

(Communicated by D. S. Beliankin, Member of the Academy, 1. VI. 1946)

The study of «vaps», to which an impetus was given by the construction of a water-power plant on the Kama River at Levshino*, is of geotechnical and scientific importance because of the specific composition, peculiar structure, which have so far not been described for Permian rocks, and their extensive distribution in the near-Kama region. The «vaps» enclosing lenses of cross-bedded fine-grained sandstones and bands of limestones, are the predominant rocks in the lower part of the Kazan stage, the so-called variegated series of rocks. Externally, they represent argillite-like «stony» pelitolites, brown, dark-brown, grey and dark-green in colour. Their average chemical composition and general characteristics have been given in an earlier note (1). The present paper contains a detailed discussion of the properties and composition of the «vaps» from the region of the sluice of the dam.

A microscopic study of plane-parallel thin-sections and polished sections has revealed that the rock consists of elastic grains, 10^{-4} to 10^{-3} cm across, cemented together with a disperse mass of an argillo-chloritic substance, carbonates and ferric hydroxide. The grains of terrigenous minerals are represented by slightly rolled feldspars and quartz, and are coated with a film of argillo-chloritic matter. Upon prolonged boiling a considerable part of the grains are readily freed from the film, but still more if the rock is treated with the Seger method. The elastic material which remains insoluble in the form of a white fine powder consisting of acid plagioclases, potash feldspars and quartz, makes up to 40 per cent of the weight of the rocks. Consequently, the structure of «vaps» is characterized by the presence of coarsely pelitic and finely aleuritic elastic particles covered with films and cemented together with the above-mentioned materials into a homogeneous, non-bedded, stone-like rock, capable of being separated into acute-angular pieces. In other words, the «vaps» possess a structure which combines the polycrystalline and the film type of cementation. A direct acquaintance with so highly distinctive a microstructure of the rocks enabled Baturin (1) to regard it as being typical of the «vaps». The present authors believe that any other name (2) of the rock described would not reflect its composition properties and peculiarities of structure.

The heavy fraction separated from the rock makes up tenth fractions of a per cent, rarely 1—2 per cent; it consists of pyrite and iron hydroxides. The quantity of transparent minerals is extremely small; among them apatite, zircon and garnet grains are encountered. The chemical composition of the «vaps» and its peculiarities are illustrated by analyses of the extreme varieties of the series, as shown in Table 1.

* 20 km north of the town of Molotov, at the point of discharge of the Chusovaya into the Kama.

From an examination of the table it will be seen that the specimens of the rock differ in the content of calcium and magnesium carbonates, loss on ignition, being similar in the closely approaching ratios: weight ratio $\text{SiO}_2:\text{R}_2\text{O}_3$, and molecular ratios, $\text{SiO}_2:\text{Al}_2\text{O}_3 + \text{Fe}_2\text{O}_3$ and $\text{SiO}_2:\text{Al}_2\text{O}_3$, showing

Table 4

	Specimen No. 1		Specimen No. 2	
	Composition, %	Passed into solution from 5% HCl, %	Composition, %	Passed into solution from 5% HCl, %
SiO_2	55.17	1.76	43.60	2.31
Al_2O_3	17.58	0.26	12.68	0.83
Fe_2O_3	1.26	1.19	5.96	1.11
MnO	0.18	0.09	0.38	0.18
MgO	3.12	0.63	7.58	4.13
CaO	3.31	2.69	11.06	10.60
K_2O	2.73	—	1.86	—
Na_2O	1.20	—	1.46	—
S	0.05	—	0.18	—
SO_3	0.15	0.13	0.16	0.17
Loss on ignition > 110°C.	6.93	—	16.58	—
Insoluble residue	—	91.68	—	67.04
Total	100.87	—	101.41	—
$\text{H}_2\text{O} < 110^\circ\text{C}$	3.78	—	2.40	—
CO_2	2.72	—	12.93	—
$\text{SiO}_2 : \text{R}_2\text{O}_3$	2.30	—	2.32	—
$\text{SiO}_2 : \text{Al}_2\text{O}_3 + \text{Fe}_2\text{O}_3$	5.9	—	5.0	—
$\text{SiO}_2 : \text{Al}_2\text{O}_3$	6.4	—	6.8	—

a uniformity of the mineral composition of the elastic material. Characteristic of the rock is a high hygroscopicity, up to 110°C. Different values of solubility in 5 per cent HCl have been found, equal to 9.47 and 32.26 per cent, in accordance with the carbonate content of the rock. Closely similar values of the insoluble part of magnesium and calcium have thus been obtained, namely, MgO 3.49 and 3.36 per cent and CaO 0.62 and 0.46 per cent. This phenomenon has been established in a large number of specimens. The passing of SiO_2 and Al_2O_3 into solution is probably due to a considerable value of the colloidal component*, equal to 13.0 per cent in the first and to 14.5 per cent in the second specimen (dried at 160°C) separated by A. I. Moshev with the centrifuge. The appearance of iron and manganese in the solution is due to the presence of their hydroxides.

The solubility of the rock in water is higher for the non-carbonate and lower for the carbonate variety, 0.11 and 0.03 per cent, respectively. At the same time it increases when the water used is saturated with 500—1000 mg/l CO_2 , giving in solution bicarbonates of alkaline-earth metals in an amount of 0.4—0.5 per cent of the rock weight. The use of a 2 per cent NaCl solution gives in the liquid phase 3.1 mg-equiv. per 100 g of rock (specimen 1) of exchange Ca and Mg.

The results of a thermal study of the «vaps» are shown by differential curves of four specimens (see figure). The curves distinctly show two groups of varieties, differing as to the composition.

The thermographic characteristics of the «vaps» compared in Table 2 with the thermal effects of pure substances (2, 4) point to the presence in the

* Less than 0.2 microns.

«vaps» of montmorillonite, chlorite, ferric hydroxide, gypsum, dolomite and calcite. No exothermal effects at 817 and 880° appeared on their heating curves because within this temperature range there proceeds a strong endothermal conversion of dolomite and calcite, disguising the exothermal effect.

Table 2

	1*	2	3	4	5	6	7	8
Montmorillonite . .	50—135°	—	—	—	612—649°	—	805—850°	—
Chlorite . .	135—161°	—	—	363—366°	390—550°	—	—	880°
Iron hydroxides . .	130—140°	—	260—328°	561°	—	—	—	817°
Gypsum . .	—	110—185°	—	375°	—	—	—	—
Dolomite . .	—	—	—	—	—	710—770°	780—920°	—
Calcite . .	—	—	—	—	—	—	825—900°	—
«Vap» . .	20—157°	210°	332°	370—435°	593—611°	740—788°	854—895°	—

* 1, 2, 3, 5, 6, 7—endothermal conversions; 4 and 8—exothermal conversions.

For the same reason the curves of the low-carbonate varieties differ from those of the carbonate ones.

The microscopic, chemical and thermal result of the study makes it possible to calculate the quantitative mineral composition of the «vaps», which is given in Table 3.

The calculation will show that the amount of feldspar and quartz corresponds to the content of elastic material, as determined after Seger. The film

Table 3

	Specimen No. 1	Specimen No. 2
Quartz	11.1	9.6
Plagioclase No. 14—19	14.4	14.1
Potash feldspar	15.5	10.6
Montmorillonite	38.8	25.7
Chlorite	12.3	10.0
Pyrite	0.1	0.8
Ferric hydroxides	1.8	2.1
Gypsum	0.3	0.4
Dolomite	2.6	17.4
Calcite	3.1	9.3
Total	100.0	100.0

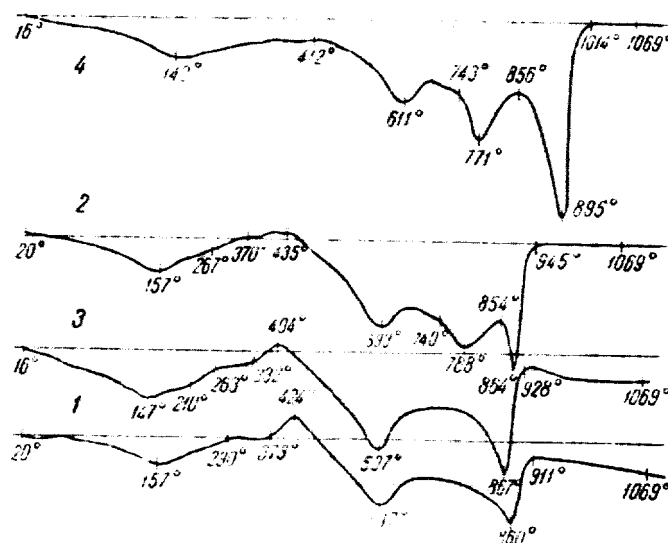
and cementing substances, represented by montmorillonite, chlorite, carbonates and iron hydroxides, when summed up give 58—64 per cent. The quantity of carbonates in the specimens varies greatly and is in full agreement with the heating curves. The absence of a dolomite terrace on curves 1 and 3 is confirmed by the low dolomite content of these specimens. The quantity of other minerals being insignificant, they are of less interest.

The composition and structure of the «vaps» are responsible for their peculiar properties. The non-carbonate and low-carbonate varieties rapidly decompose upon being immersed in water after the loss of their natural moisture, and also upon freezing, while carbonate varieties are comparatively resistant and are devoid of this property.

The peculiar physical properties of «vaps» are dependent on their high content of montmorillonite in a state of aged colloid. When losing its natu-

ral moisture, the montmorillonite contracts (³) and microfractures appear in the rock. When immersed in water in such a condition, unequal expansion and wedging of the rock into small pieces takes place along irregular curvilinear surfaces. A similar unbalanced state sets in when the rock is immersed in water after freezing, when part of the water is likewise removed. In either case the structural bond is disturbed, which is attended by disturbances in the monolithic character of the rock.

The geotechnical properties of the rock are likewise dependent on its composition and structure. According to K. S. Naumenko, the compression



of the «vaps» under load obeys two laws: at first the compression proceeds at a high rate, then it becomes much slower.

From the aforesaid it may be seen that the «vaps» is a finely elastic rock of a rather peculiar composition and structure. The strength of the rock varies and is symbiotic to the quantity of carbonates.

The formation of the rock must have proceeded by way of a slow orthokinetic coagulation of the diverse particles of montmorillonite, chlorite and iron sols on grains of the settling suspension of feldspars and quartz. The settled mass was enriched in carbonates, usually carrying a positive charge (⁴), which favoured the consolidation of the sediment. The low-carbonate varieties were probably formed at the periods of less intense carbonization. The possibility is not excluded that the rock might have lost a considerable part of carbonates in the course of weathering which proceeded non-uniformly throughout the sediment.

The V. M. Molotov State University,
Molotov.

Received
1. VI. 1946.

REFERENCES

- ¹ A. M. Kuznetsov and D. P. Prochukhan, C. R. Acad. Sci. URSS, 26, No. 3 (1949).
- ² V. A. Perederiev, *ibid.*, 49, No. 3 (1945).
- ³ D. S. Beliankin and V. P. Ivanova, *ibid.*, 18, No. 4-5 (1938).
- ⁴ I. F. Бегр, А. В. Илюков и Е. Я. Род, Термостат, изд. АН СССР, 1944.
- ⁵ S. Yussupova, C. R. Acad. Sci. URSS, 51, No. 8 (1946).
- ⁶ В. М. Горникова и Г. Осташенко, Родн. журн., 1, No. 4 (1935).

PALAEONTOLOGY

ON THE GENUS *HEMIFUSULINA* MOELLER

By S. ROSOVSKAYA

(Communicated by I. I. Schmalhausen, Member of the Academy, 16. IV. 1946)

In 1877 Moeller⁽²⁾ separated out the family *Fusulinidae* and four genera: *Fusulina*, *Schwagerina*, *Hemifusulina* and *Fusulinella* on account of the fact that *Hemifusulina* carries all the features of the genus *Fusulina*, differing from it chiefly in double partitions. The author cited above believed that in representatives of the genera *Hemifusulina* and *Fusulina* the walls had a similar structure and were intersected by numerous closely spaced pore channels.

Having ascertained that the genotype of the genus *Hemifusulina* of Moeller had actually no diaphanotheca, Schellwien proposed to consider *Hemifusulina bocki* Moell., the single species of the genus *Hemifusulina*, as a species of the genus *Fusulina*⁽³⁾.

The Chinese palaeontologist Lee studied in 1933—1937 the *Fusulinidae* from the limestones of the Carboniferous of the Donetz basin, including forms corresponding to *Hemifusulina bocki* of Moeller or *Fusulina minima* of Schellwien. From the presence in the latter of non-folded septa, thick chomata and a keriothecal structure of the wall, Lee arrived at a conclusion as to these forms belonging to the genus *Triticites* Girty⁽⁵⁾. Having discovered in this group of *Fusulinidae* specimens with a thicker wall than that of *Fusulina minima*, the author thought it possible to found new species, giving them the names of *Triticites ellipticus* and *Triticites graciosus*.

In 1939 Brazhnikova described from the limestones M_6 and N_1 of the Donetz basin *Triticites ellipticus* Lee and *T. graciosus*, pointing out the absence of diaphanotheca and tectoria, and the presence of distinctly pronounced keriotheca. The peculiarity in the theca structure of these species, as compared to the other *Triticites*, lies in that they show no «splitting» of trabecules (lamellae of shell between alveoli) in two towards the tectum, which is a feature characteristic of the keriotheca of other *Triticites*⁽¹⁾.

The finely alveolar structure of one-two outer volutions has also been mentioned by Putrja, who studied the forms in question in 1940 from the limestones M_6^2 — N_1^1 in the Donetz basin, and named them *Fusulina minima* var. *graciosa* Lee⁽³⁾. In his description the author refers to the presence of tectum, diaphanotheca and two poorly developed tectoria in all the inner whorls. Putrja identifies his *Fusulina minima* var. *graciosa* with *Fusulina minima* Schellw., *Triticites ellipticus* Lee and *T. graciosus* Lee, described both by Lee and by Brazhnikova.

In 1940 Rauser-Chernousova, studying *Fusulina minima* from the Middle Carboniferous of the Samara Bend, included into the synonymy *Hemifusulina bocki* Moell. and *Fusulina minima* Schellw.⁽⁴⁾ Referring to the structure of the theca, this author points out that in the form described by her the structure is very fine, and generally the differentiation of the theca is hardly

distinguishable. No diaphanotheca in the middle whorls is perceivable, unless the specimen is in a good state of preservation. In the outer whorl a very finely alveolar keriotheca is generally discernible. In comparing her species with the forms described by Schellwien ^(*) and Moeller ⁽²⁾, Rauser-Chernousova writes that the identification of this form with *Fusulina minima* Schellw. presents no difficulties, and that to the full description made by Moeller only somewhat larger dimensions and a slightly thinner theca should be added ^(*).

On the material of the group *Fusulina minima* from Middle Carboniferous deposits of the Moscow basin studied by the present writer (Kashira, Podolsk and Myachkovo horizons) it proved impossible to observe either diaphanotheca or tectoria. The forms of this group have a wall consisting of two layers, with keriotheca in all whorls. The thickness of the wall of the species distributed in the Myachkovo beds in the last but one revolution is 50—60 μ , while the forms characteristic of older strata have a considerably thinner theca



Fig. 1. Scheme of wall structure in *Fusulina* (A), *Triliticites* (B) and *Hemifusulina* (C). a, outward and inner tectoria; b, tectum; c, diaphanotheca; d, keriotheca.

(25—30 μ), for which reason the keriotheca in the inner whorls of most specimens is hardly discernible.

A detailed study of *Fusulinidae* carried out in the recent years has shown that in some forms, along with well-pronounced diaphanotheca in the inner whorls, an alveolar structure of theca in the outer ones is observed. Putrja founded two more genera from the upper part of the middle and lower part of the Upper Carboniferous of the Donetz basin, namely, *Pseudotriticites* and *Prottrititices**, the representatives of which have a wall with a finely alveolar diaphanotheca in the first three or four inner revolutions and with a three-layer structure (keriotheca, tectum and tectoria) in the subsequent ones ⁽³⁾.

Starting from the fact that the microstructure of the theca is a very important generic feature, the present author cannot agree to the suggestion to refer the forms here considered to the genus *Fusulina* Moell., characteristic of which is a four-layer structure of the wall with the presence of diaphanotheca, tectum and two tectoria.

Their difference from the representatives of the genus *Pseudotriticites* is also evident, since even in the early whorls diaphanotheca is wanting, and the wall always consists of two layers.

In the present writer's opinion it is impossible to refer these forms to the genus *Prottrititices* Putrja, since the representatives of this genus have a theca of four layers with a porous structure. The diaphanotheca in the representatives of this genus is easy to trace in the inner whorls.

The description of the above forms as belonging to *Triliticites* seems also doubtful, since the keriotheca itself differs rather considerably from the typical keriotheca of *Triliticites* in much wider trabecules divided by very narrow lighter pores, which is often responsible for an apparently amorphous nature of the wall and impedes the study of its microstructure.

On the writer's material is also confirmed the peculiarity reported by Brazhnikova for *Triliticites ellipticus* and *T. graciosus* which consists in that, in contradistinction to forms having a typical keriotheca, the trabecules

* The description of this genus is not known in the literature, unpublished data being available.

in the *Fusulinidae* named are not thickening towards the centre, and the alveoli are not branching.

As already mentioned, the wall structure of the shells is the most important systematic and evolutionary feature of *Fusulinidae*. Therefore all the afore-said induces the writer to unite in one genus both the forms studied by her in the Moscow basin and the forms from other localities, described and referred by other authors to different genera, and to restore the genus *Hemifusulina* Moell., introducing somewhat greater precision into the diagnosis.

It is essential to point out that the insufficient knowledge of the structure of the walls of the shells, owing to which the forms in question were referred to two different genera, *Fusulina* and *Triticites*, has led to a considerable stratigraphic confusion and wrong ideas concerning the distribution of these latter genera. As a matter of fact, the two genera are of guiding significance: the disappearance of representatives of the genus *Fusulina* and the appearance of the genus of *Triticites* is a reliable indication of the Middle Carboniferous deposits being substituted by Upper Carboniferous ones. Moreover, the genera *Fusulina* and *Triticites* are members of different phylogenetic branches. The ancestors of *Triticites* were *Fusulinella* and *Protriticites*, and those of *Hemifusulina*, probably *Fusulina*. The two branches developed in a parallel way, with a number of similar characters, but with some essential distinctions, as already shown above.

The present paper contains a description of a typical species occurring among other forms of the same genus in the Moscow basin.

Family *Fusulinidae* Moell.

Subfamily *Schwagerininae* Dunbar et Henbest

Genus *Hemifusulina* Moell., 1877

D i a g n o s i s. Shell small, ovoid in shape. Spiral closely coiled in the inner whorls, becoming higher and more uncoiled in the outer ones. Theca thin, made up of two layers, consisting of tectum and keriotheca. Trabecules wide, divided by very narrow parallel pores. Septa from slightly to moderately folded, forming small arches, rounded in shape and regularly situated. Chomata rounded, well developed in all whorls. Axial deposits lacking. Aperture single, regularly situated.

N o t e. Distinguished from *Fusulina* by the absence of diaphanotheca; from *Triticites*, by the presence of branching alveoli. Genotype: *Hemifusulina bocki* Moeller.

Hemifusulina bocki Moell.

1878. *Hemifusulina bocki* Moeller (2). 1908. *Fusulina minima* Schellwien (3). 1936—1937. *Triticites ellipticus* Lee (5). 1939. *Triticites ellipticus* Brazhnikova (1). 1940. *Fusulina minima* Rauser-Chernoussova (4).

Shell oval in shape. Ends obtusely rounded. This form is rather constant, beginning with the young stage of development. *L:D* from the third to the last volution varying within the range of 1.96 to 2.35. Surface of shell covered with shallow septal furrows. Dimensions small: *L*=2.36 to 3.0 mm; *D*=1.1 to 1.4 mm. Forms attaining *L*=3.9 mm and *D*=1.7 mm are less frequent. Number of volutions 6½ to 8.

Initial chamber small. Its diameter is from 50 to 95 μ. Theca thin, made up of two layers, consisting of tectum and keriotheca. Its thickness in the volutions preceding the last one is 46 to 67 μ. Trabecules wide, divided by narrow unbranching pores.

Septa approximately of same thickness as theca, regularly folded, forming small arches of rounded or subsquare shape. Their height is half the lumen of the chambers. In the axial ends of the last three volutions they form an alveolar network.

Aperture rather narrow, regularly situated. Tunnel usually X-shaped. Its breadth is equal from $\frac{1}{10}$ to $\frac{1}{11}$ of the length of the corresponding volution. Height about half the lumen of the chambers.

Chromata well pronounced to the last volution, rather massive, rounded in shape.

Comparison. The specimens available to the author do not differ materially from the *Hemifusulina becki* described by Moeller. An insignifi-



Fig. 2. *Hemifusulina becki* Moell. a. Moscow basin, Peski. Axial section. $\times 17.5$. Middle Carboniferous, horizon C_{II}^4 ; b. Moscow basin, Dubovo. Axial section. $\times 17.5$. Middle Carboniferous, horizon C_{II}^4 .

cant distinction consists in somewhat larger dimensions of the shells of the forms from the Moscow basin and by a thicker theca. For the largest specimens Moeller gives the following dimensions: length, 2.6 mm; diameter, 1.2 mm.

Occurrence and age: the Moscow Basin, the Middle Carboniferous. Horizons C_{II}^2 , C_{II}^3 , C_{II}^4 .

Received
16. IV. 1946.

REFERENCES

- ¹ И. Е. Браунников, Тр. Геогр. общ. АН УРСР, VI (1929).
- ² В. — Моеллер, Матер. геол. России, VIII, 1878.
- ³ Ф. С. Гутроп, Матер. по геол. и палеон. азиат. Чер. Геол. учреждение, сб. 11, 1940.
- ⁴ Д. М. Пауэрс — Чепмен, Тр. Геогр. общ. АН УРСР, VII (1940).
- ⁵ S. Lee, Bull. Geol. Soc. China, XVI (1936—1937).
- ⁶ F. Schellwien, Palaeontogr., 55, 1903.
- ⁷ V. Moeller, Neues Jahrb. Miner., on Geol., 116 (1877).

GENETICS

THE EFFECT OF POLYPLOIDY UPON THE TAPETUM IN POPPY

By E. N. VOLOTOV

(Communicated by L. A. Orbeli, Member of the Academy, 16. IV. 1946)

Certain, more often specialized, tissues of diploid animals and plants are known to be composed of tetraploid and even octoploid cells, *i. e.* of cells containing nuclei with double and quadruple chromosome complements. In plants such are the cells of the tapetum.

Studies made on certain experimentally produced polyploid plants have shown that their cells and tissues do not answer in a similar way to polyploid condition: the rate of growth, the relative size and the number of cells in a given organ vary in a different way in the different tissues of the same plant (^{1,2}).

In the present paper are communicated the observations made by the writer on the effect of polyploidy on the tapetum cells in the anthers of poppy which are normally, as pointed out above, polyploid in diploid plants. The work has been carried out with the aim to obtain new information on the regularities governing the development of polyploids, and additional comparative data on diploids and polyploids. The observations were mostly made on the tapetum cells of the anthers of the diploid *Papaver somniferum* L. and of its tetraploid form which had been obtained by the writer with the aid of colchicine (³).

Tapetum cells were studied in all cases at the same stage corresponding to the 1st meiotic division. This stage represents in poppy a period beyond which no mitoses can be observed in the tapetum cells. Observations were made partly on permanent preparations stained with gentian violet, partly on temporary aceto-carmin preparations. While preparing the latter, the content of the anther was squeezed out and the pollen mother cells, as well as the tapetum cells, were inspected separately from other tissues. In either case the material was fixed with the Carnoy fluid.

The diploid *Papaver somniferum* contains in its cell 22 chromosomes, whereas the tetraploid form has 44 chromosomes.

Cytological investigation of the anthers of the diploid *Papaver somniferum* has shown that in the tapetum cells, when a single metaphase plate is present, no more than 88 chromosomes can be counted, that is, there is present an octoploid complement. When two nuclei are present in the tapetum cells (such is the case in the great majority of cases) sometimes simultaneous division may be observed in the two nuclei; in such cases in each of the metaphase plates no more than 44 chromosomes are observed, the cells being again octoploid.

Aside from *Papaver somniferum* I have studied other species of *Papaveraceae*, namely: *P. paeonium* Schrenk. ($2n=12$), *P. alpinum* L. ($2n=14$), *P. rhoeas* L. ($2n=14$), *P. nudicaule* L. ($2n=14$), *P. hybridum* L. ($2n=14$), *P. latericum* C. Koch ($2n=14$), *P. macrostomum* Boiss. et Huet ($2n=14$), *P. commutatum* Fisch. et Mey. ($2n=14$) and other species of different genera

of the family, namely: *Roemeria n fracta* (Stev.) D. C. v. *setosa* ($2n=14$), *Argemone mexicana* L. ($2n=28$), *Escholtzia californica* Cham. ($2n=12$), *Chelidonium majus* L. ($2n=42$), *Dicentra spectabilis* D. C. ($2n=16$).

In all the species enumerated above the tapetum cells in the anthers proved to be octoploid, i. e. with 12 somatic chromosomes the tapetum cells were found to have 48 chromosomes, with 14—56, with 16—96, with 28—112. In other words, in all the diploids belonging to the family *Papaveraceae* the ratio of the chromosome number in somatic cells, for example, in the root tips, to that in the tapetum cells was 1:4.

One should naturally expect to find in the tapetum cells of the experimentally produced *Papaver somniferum* a doubled chromosome number, i. e. 16-ploid cells.

As shown by cytological investigation, no such doubling took place, however. Thus, in the experimentally produced tetraploid *Papaver somniferum* the tapetum cells had 88 chromosomes, i. e. the same number as in the diploid *P. somniferum*. The ratio of the chromosome number in the somatic cells and in the tapetum cells was thus found to be 1:2 in the tetraploid.

This fact shows that the doubling of the chromosome number in the soma of the diploid does not necessarily involve all the tissues, and does not affect, in particular, the tapetum cells.

What are the reasons responsible for the absence of the octoploid cells in the tetraploid poppy? Is it a result of an excessive charge of the cells with chromosomes, and may not octoploid chromosome complement be a threshold of viability similar to that observed in the majority of plants in which even tetraploidy is sometimes found to lead to a number of alterations resulting in lethality?

A partial answer has been obtained by the author as a result of a study carried out on the multi-chromosome species of *Papaver*: *P. orientalis* L. ($2n=42$) and *P. dubium* L. ($2n=42$). The tapetum cells in these species have been found to contain 168 chromosomes, thus being octoploid, the ratio between the chromosome numbers in the somatic and tapetum cells being the ordinary one, i. e. 1:4, in spite of the great chromosome charge. The study of these representatives of *Papaveraceae* made it seem doubtful that the octoploid complement is a limit for the existence of the cells of *P. somniferum*, since the expected 16-ploid tapetum cells of the experimental tetraploid *P. somniferum* should contain 176 chromosomes, i. e. only 8 chromosomes more than in the two species mentioned above.

It is well-known, however, that the threshold of polyploidy differs as to the species: some plants may retain viability with octoploid chromosome complements, others do not stand even tetraploidy. The existence of two species possessing large chromosome numbers in their tapetum cells should not be taken to prove anything in other species of the same family. There is, however, direct evidence supporting these doubts; thus, J. P. Mirinda (unpublished) has found in the tapetum of the diploid *Papaver somniferum* one unquestionable case of 16-ploid chromosome complement. This case, which is probably an extremely rare exception in diploid poppy, must be the general rule in tetraploids. One is compelled to think that the absence of 16-ploid tapetum cells in tetraploid poppy is to be explained by something else than an excessive chromosome charge.

Some light upon the problem might be shed by the experimentally produced tetraploid poppy with low chromosome numbers; they might support (or reject) the hypothesis as to the new quantitative chromosomal relations in the tapetum cells and other somatic tissues without any limit for the chromosome charge.

Unfortunately, the writer was so far unable to obtain a tetraploid in any low chromosome *Papaver* species; he has found, however, an alleged spontaneous tetraploid obtained from the Erevan Botanical Gardens under the name *Papaver nudicaule* L. f. *gigantea*.

The form in question had increased (approximately, twice) flowers, leaves, flower-stalks, seeds, seed capsules, hairs upon their leaves and buds and pollen grains. These plants set very few seeds, a great proportion of which is empty. A cytological study carried out by the writer has shown that there were 28 chromosomes in the soma instead of the 14 typical of *P. nudicaule*. The study of meiosis has revealed 3—5 quadrivalents. This form was thus found to be a typical autotetraploid, both after its habitus and its meiotic behaviour. The study of the tapetum cells has revealed 56 chromosomes, *i. e.* they were octoploid like in the diploid *P. nudicaule* (see above); thus, the ratio of the chromosome number in the soma to that in the tapetum cells was found in this alleged autotetraploid to be identical with that in the experimentally obtained tetraploid of *P. somniferum*.

Finally, we have studied the tapetum cells in hybrids. There were studied F_1 of two hybrid combinations: tetraploid *Papaver somniferum* ($4n=44$) \times *P. orientale* ($2n=42$) and *P. somniferum* ($2n=22$) \times *P. orientale* ($2n=42$). The first named hybrid had in its tapetum cells 86 chromosomes; the other, 66 chromosomes. Consequently, the two hybrids had the same ratio of chromosome numbers in the somatic and tapetum cells as those in the experimental autotetraploid and the alleged spontaneous autotetraploid, *i. e.* 1:2.

A characteristic feature common to all the observed cases of such ratio (1:2) was that it occurred only in newly formed systems, both in hybrids and autotetraploids. This is comprehensible in the case of hybrids where we are confronting a special mode of regulation leading in the course of the development to diverse differentiation of the tissues and organs, unlike the parental ones, owing to new genetic combinations and to new interactions leading in some cases to a physiologically unbalanced condition and even to unviability. On the contrary, cases of autotetraploids in which we find only doubling of the genetic material without any disturbance in its balance stand in need of analysis. The most probable explanation was offered by Randolph and Hand (⁴) who connected the alterations taking place in autotetraploids with the cumulative effect of the genes.

Adopting this point of view, one may imagine some of the alterations caused by autotetraploidy, such as chemical changes, consisting mainly in a changed quantity of the components, morphological, represented mainly by a changed rate and number of cell divisions, not as a result of a simple additive action of the genes, but as a kind of differential action in the morphogenesis of each tissue, of each chemical process in the cell metabolism, caused by the cumulative effect of definite genes. We are thus brought to believe that the genes which determine or regulate the chemical composition of the organism and the formative processes in the developing organs do not act in the same way in diploids and tetraploids. An instance of such differentiated effect upon the chemical composition is found in the tetraploid ryegrass where, with a considerable increase in sugar content, there was found only an insignificant change in nitrogen content (⁵), and also in the tetraploid tobacco where, along with an 18—33 per cent increase in nicotine content and a considerable increase in nitrogen, fats, calcium and potassium, there was found a decrease in carbohydrates, sulphur and phosphorus (⁶). An example of morphological alterations may be found in the tetraploid maize where the endosperm cells are increased 3.5 times, whereas the other cells—only 2 times (²), and in *Physalis* where the growth of the stamens in the tetraploid form is behind that in the pistils (⁷).

It is obvious that in the effect of autopolyploidy one should distinguish instances of direct influence of increased nuclei from those connected with the increase in the cell dimensions, such as in the case of pollen grains, stomata, hairs, meristematic cells, although such a distinction cannot be followed very rigorously.

The case of tapetum cells discussed in the present paper probably belongs also to those of cumulative gene action. Here the change has affected not

only the number of chromosomes, but also the number of the tapetum cells. The counts of tapetum cells made on permanent slides showed one median section of the anther to contain on the average 90 tapetum cells in the diploid poppy and only 52 in the tetraploid. The possibility is not excluded, therefore, that the basic function of the tapetum, the nutrition of the pollen mother cells, may affect the fertility as a result of such a strong reduction in their number. The alteration in the tapetum together with a number of other alterations, namely, morphological variation in the organs of the flower, which may lead, like in *Physalis*, to a changed mode of pollination (?), chemical changes leading, for example, to an altered odour of the flowers and to a changed frequency of visiting by insects (according to the writer's observations, the flowers of the tetraploid snapdragon and poppy are less visited by bees and humble-bees), show that individual alterations depending on cumulative gene action, which at first seem insignificant, may lead to very important changes in the organism as a whole, to a changed physiological balance in the organism, to morphological and functional changes which are of import in the struggle for life. In some respects they resemble changes caused by hybridization, which may likewise be illustrated by the case of the tapetum. These new relations, their scope and amount, depending both on the quantity and quality of the genes involved in the initial heterozygous material, constitute an important source for artificial and natural selection in autotetraploids.

Institute of Cytology, Histology and Embryology,
Academy of Sciences of the USSR.

Received
16. IV. 1946.

REFERENCES

- ¹ J. S. Traub, Biol. Zbl., H. 11/12, 657 (1940). ² L. E. Randolph, Symposium on theoretical and practical aspects of polyploidy in crop plants, Biol. symposia, IV, p. 145—169, 1941. ³ E. N. Vekotov, C. R. Acad. Sci. USSR, 31, No. 2 (1944).
- ⁴ L. E. Randolph and D. Hand, J. Agr. Res., 60, 51 (1940). ⁵ J. T. Sullivan and Wm. Myers, J. Am. Soc. Agron., 31, 869 (1939). ⁶ Y. Noguchi, H. Oka and T. Ochiai, Jap. J. Bot., 10, 543 (1940). ⁷ E. H. Borodovskii, B. H. Gurovskii, Рефераты работ зарубежных бот. наук АН СССР за 1941—1943 гг., стр. 282.

ECOLOGY

**CROP PLANTS UNDER EXTREME CONDITIONS OF HIGH
ALTITUDES OF THE NORTH CAUCASUS AND THE NORTH-EAST
ALTAI**

By A. A. MALYSHEV

(Communicated by A. A. Richter, Member of the Academy. 15. IV. 1946)

In our experiments on the cultivation of crop plants in different highly elevated zones of the North Caucasus (Teberda reserve) and the North-East Altai (Altai National Park) we were able to investigate the peculiar character of the development and growth of a number of cryophile crops in the surroundings which set limit to their viability.

It has been found that the zone of the climatic barrier of the cultivation of crop plants in Alpine region coincided on the whole with the upper boundary of the forests, and extends some 100—200 m higher, along with the penetration of single individual trees; in the North-East Altai, for example, it extends so far as to the mountain tundra belt. This zone is characterized in the two geographic regions named by an average temperature of the growing period of 8—10°C. by a sum of temperatures over 10° (counting from zero) totalling 500—700°, and by the duration of the frost-free period (in the meteorological chamber) equal to 80 days. In spite of the fact that the frost-free period is limited (at the surface of the soil) by 30—40 days, a number of crop plants in them can usually grow during 2½—3 months in connexion with the «chilling» which takes place in them.

The amount of precipitations during the four summer months in this zone is 600—700 mm, more than twice that of the zone 1½ km below.

In the western part of the northern slope of the Main Caucasus Range the boundary of the forest runs at an altitude of 2400—2500 m. In the part of the Altai considered here, in the region of the junction of three mountain systems—the Altai, the Kuznetsk Alatau and the West Sayan, the upper limit of the forest is at an altitude of 1700—1900 m. In the last named region the highest limit of our crops was in 1944—1945 at an altitude of 1750 m, at the upper boundary of subalpine thin forest, and at an altitude of 1850 m at the summit of the mountain Korbulu, in the mountainous tundra belt. In the North Caucasus the experiment plot was located (1940—1944) on the one of the summits of the mountain ridge Mukhu-Bashi, within the Alpine belt, at an altitude of 2700 m. According to the literary data (2), the altitude of 2700 m is the highest limit of agriculture in Europe (Spain, Sierra-Nevada mountains). How severe are the conditions at an altitude of 2700 m in the Caucasus is shown by the fact that the snow does not melt everywhere near the experiment plot during 11 months.

As a result of the experiments in the zone in question in the North Caucasus we succeeded in growing radish of normal size, turnip weighing 140—200 g each, cole-rape weighing 60 g each, peas (variety «Dwarf») forming green pods, onion forming green leaves, garlic, potato yielding up to 230 g of tubers per hill. The variety «Epicur» yielded individual tubers weighing 100 g each.

In the mountain-tundra belt of the North-East Altai (1850 m) we obtained: radish of normal size, green onion, peas with green pods, and potato yielding up to 100 g of tubers per hill; individual tubers were 50 g each. At the upper limit of the forest (1750 m) in the Altai there were grown: radish weighing 30 g each, turnip 70-130 g each, peas («Dwarf») forming normal but underdeveloped seeds, broad bean forming green pods, garlic and onion for leaves, flax and buckwheat forming immature seeds, potato yielding up to 35 g per hill. Individual tubers («Berlichingen», «Lorch», «Early Rose») weighed 130 g each. Grains (winter rye, wheat, barley, oats) formed ears at all the plots, and in the Altai (1750 m) they were covered with snow at the phase of blooming.

Aside from the obstacles of climatic character, the cultivation of crop plants at high mountain zones is hindered by the edaphic factors. The high acidity of the Alpine-meadow soils of the Alpine type, and especially of the mountain-tundra soils of the mountain zone in the Altai necessitates lime treatment. The reaction to this measure was especially striking in the case of onion, radish, turnip, buckwheat, barley, wheat, which showed scarcely any development without introduction of lime.

The effect of fertilizers was also very great on the soils of the Alpine type. The height of the culms of grains (barley, wheat) and of the stems of the legumes under the application of fertilizers and lime at an altitude of 1750 m (the Altai) was twice as great as in the case of untreated soil.

In the experiments in the North Caucasus under identical conditions (full mineral fertilizers) at plots located at different altitudes, the energy of tillering of grain crops and the formation of tubers by potato invariably increased with altitude. Putting the energy of tillering of the grains at an altitude of 1350 m at 100, we had 122 per cent at an altitude of 2350; at 2700 it was 188 per cent. Naturally enough, the number of sterile culms increased with altitude, being 14 per cent at the lowest plot and 61 at the uppermost one.

The height of the main culms in the grasses at a 2700 m elevation was only 60 per cent that of the lowest plot.

The average number of tubers per hill (variety «Imandra») was at the uppermost plot 240 per cent to that of the lowest one, while the weight of the tubers was only 23 per cent to that yielded at the lowest plot.

Branching of the stems of sunflower is likewise found to increase with height with a striking depression of the main stem (the Altai), a formation of several heads of cabbage on the common root (the Caucasus), *et c.*

One should think that being deprived of the possibility to use the assimilation products under the extreme conditions of development or growth of the main aboveground organs, the crop plants spend the formative materials for building additional vegetative organs, which is easier under such conditions. In the process of their ontogenesis the crop plants undergo a sharp change in their habitus under the conditions of high altitude and become, as is the case of grasses, adpressed, strongly tillering and with strong antocyanine coloration of the culms and leaves. The ability of the crop plants to be modified under extreme surrounding conditions is manifest also in an increase in frost resistance with altitude. At an elevation of 2350 m (the North Caucasus) the aboveground part of the girasole could stand a frost of -4.8° at the level of the soil and -10.0° in the air, and died together with the Chinese cabbage only at -6.8° on the surface of the soil and -12.0° in the air. At an elevation of 1750 m (the Altai) the leaves and stems of potato could stand without damage -4.0° at the surface of the soil, and at 1850 m it died only in part at -6.0° on the surface of the soil.

The growing period of the crop plants usually increases with altitude. The peas «Dwarf», which showed at an altitude of 1350 m (the Caucasus) a length of the phase «seedlings—formation of pods» of 39 days, had increased this phase to 66 days at an altitude of 2700 m. Wheat «Garnet» showed,

respectively, an increase in the phase «seedlings—earings» by 26 days. At the same time in the wild aborigens there is observed a shortening of the growth period with altitude. Thus, *Anemona altaica* Tisch. at an altitude of 470 m (the Altai) passes the phase from the beginning of growth to flowering within 24 days (1944), whereas at 1750 m the Alpine ecotypes of the same species pass this phase within 7 days. *Trollius asiaticus* Z. showed a duration of the same stage at 470 m of 37 days, while the Alpine ecotypes passed it at 1750 m within 14 days.

The conclusion is very plausible that the lengthening of the stages of development of crop plants should take place only during the first years of their cultivation in the Alpine zones, until the plants are adapted to the unusual surrounding conditions⁽³⁾. In further generations reproduced at the same zones, there will take place in the course of adaptation a change in the hereditary properties of the plant, in particular, it will develop earliness.

In the work by Baranov⁽¹⁾ we find indications to the effect that barley shows an increased earliness after repeated reproduction at highly elevated zones of the Pamir.

This opposite character of the variation of developmental processes of the cultivated and wild plants with altitude clearly demonstrates the dialectics of the natural phenomena. A similar collision is observed at highly elevated zones in the variation of the exchange in crop plants and wild vegetation. According to Grebinsky⁽⁴⁾, cultivated plants show at first a more intense production of carbohydrates, in particular, of sugar, whereas wild plants, on the contrary, display a weaker carbohydrate reaction. In the course of adaptation in the mountains cultivated plants gradually become able to pass to a normal exchange approaching the type of redox processes, characteristic of the wild aborigens of those zones.

It stands to reason that the growth of crop plants should be considered not only in connexion with its depression caused by unsuitable surroundings, but as an active biological process taking place during ontogenesis and leading in the course of generations to a more profound alteration in the nature of the plant organism.

The study of the problem of cultivation of crop plants at high altitudes, under conditions typical of the limit of their distribution, is necessitated by the advancement of farming to high-mountainous regions. Of course, successful introduction of crop plants to the high-mountainous zones is dependent on the development of agricultural technique. However, growing crop plants above the limit of the forest, for example, within the area of elevated places, such as exist in the Altai and the Caucasus, will require hot beds and greenhouses which are likely to be highly efficient under the conditions of intense sun radiation available in the highlands.

Received
15. IV. 1946.

REFERENCES

- ¹ И. А. Баранов, Растение и среда, изд. АН СССР, 1940. ² Г. В. Ковалевский, Природа, № 3 (1937). ³ Т. Д. Лысенко, О наследственности и ее изменчивости, 1944. ⁴ С. О. Гребинский, Диссертация, Б-ка Каз. гос. ун-та.

CYTOLOGY

OBSERVATIONS ON THE NUCLEUS AT INTERKINESIS

By G. V. KHARLOVA

(Communicated by I. I. Schmalhausen, Member of the Academy, 3. V. 1946)

In a note published in this periodical ⁽¹⁾ Makarov reports on having succeeded in «fixing» paranekrotic changes in cells; he states that, by applying various agents, he has obtained specific reactions to each of them, and, furthermore, that all the phenomena observed were reversible. Makarov describes, in particular, the effect of hypertonic solutions upon the content of the nucleus. Under the influence of hypertonic solutions (glucose, etc.) and after fixation he observed fine granulation appearing in the nucleus, which could be observed ultramicroscopically. The granulosity disappears as soon as the influence of the agent is removed.

Our experiments, arranged with the aim of studying the condition of the nucleus at interphase under application of a nearly identical technique, furnished results which will be briefly summarized in the present communication.

During the last years conclusive data have been obtained leading to a radical revision of our ideas as to the structure of the nucleus. Many authors have more than once advanced the idea ⁽²⁾ that the nucleus consists solely of chromosomes. The nucleus is a dynamic structure characterized by its cycle. What was earlier called the «resting nucleus» is now regarded as one of the phases of the complex cycle during which the chromosomes are in a state of stretching and, probably, are highly hydrated. The invisibility of the chromosomes *in vivo* is explained by an identical coefficient of refraction of all the materials composing the nucleus. On slides fixed and stained after Feulgen no chromosomes can be discovered in the nucleus. According to number of investigators, this should be attributed to the fact that the chromonema contains no nucleic acid.

Our aim was to study the condition of the interkinetic nucleus under the influence of various solutions. In contrast to the majority of the previous studies ^(5,6), we have studied the alterations occurring in the nucleus not merely *in vivo* but mainly on fixed preparations stained after Feulgen. This method was recently applied by Painter ⁽³⁾. It was known that with an increase in the pH toward slightly acid and hypertonic solutions cause coarser structures to appear in the nucleus, the appearance of granules clearly visible *in vivo*. These phenomena are considered to be an example of reversible gelatinization, since when the nuclei are embedded into a normal medium they become optically empty again. It has not been explained what are the gelatinized structures, and what is their relation to chromosomes.

We have suggested that in the case the chromosomes are really in a swollen state, they may be seen when dehydrated. Bits of tissues were treated with the reagent and then fixed in Zenker's or Flemming's fluids and stained, mainly with the Feulgen's stain. In spite of the recent criticism ⁽⁴⁾,

this method is generally considered to be an ideal one for discovering in the cell nucleoproteids of the type of desoxiribose.

The object of our observations was the hermaphroditic sex gland of *Helix pomatia*, which has more than once served for experiments. This communication is concerned only with experiments carried out with the following solutions:

1. Hypertonic solutions: a) Ringer's solution with double and quadruple amount of NaCl; b) glucose; 3-molar solution.
2. Hypotonic solutions: a) Ringer's solution diluted twice with distilled water; b) distilled water.
3. Solution of ammonium chloride, 0.4 per cent.

Our attention has mainly been attached to alterations taking place in the nuclei of the connective tissue between the canals of the hermaphroditic gland. On the control preparations fixed and stained after Feulgen were



Nucleus of a cell from the connecting tissue. Fixed with Flemming's fluid, stained after Feulgen. a, normal condition; b, Ringer + 2 NaCl, 30 min.; c, 0.4 per cent NH_4Cl , 20 min.; d, 4 NaCl, 30 min.; e, distilled water, 30 min.

visible fine granules of nearly equal size, which represented typical granules of chromatin (Fig. a).

After the portions of the tissue had been immersed into the hypertonic solution consisting of the Ringer's fluid with double amount of NaCl and particularly with quadruple amount of this salt, the nucleus appeared to be somewhat shrunken because of dehydration; the granules of chromatin appeared as it were fused into bead-like structures, well stained with Feulgen (Fig. b).

These granules were typical chromomeres and the bead-like structures were nothing else than chromosomes. Sometimes they were bent and looked like loops, sometimes they were rather long and stretched across the nucleus. Occasionally the chromosomes could not be seen at all. The remaining contents of the nucleus, when stained after Feulgen, takes a diffuse light violet stain.

As a result of a treatment with 3-molar glucose and with the NH_4Cl solution the bead-like structure could also be clearly seen (Fig. c).

After the fragments of the tissue were treated with hypertonic solutions and in particular with NH_4Cl solution, no more chromatin granules were visible on preparations obtained from the treated fragments, but rather regular chromosomes. That there were not simply coagulated granules of some enigmatic nuclear contents, as supposed by Makarov, but the chromosomes, is shown by the following observations. In such cases where under the influence of the treatment the nuclear membrane was demolished, free chromosomes could be seen lying in the cytoplasm of the cell (Fig. d).

The chromomeres are lying farther apart on such preparations as compared with the case of intact nuclei. It is possible to see under the highest power extremely thin connecting junctions. At the same time the free ends of the

chromosomes can be seen. Whether they are really free ends, or merely points of fracture, could not be decided.

A nucleus subjected to the action of hypertonic solutions (Fig. *e*) displays the following alterations: its size becomes greater, which points to swelling; the diffuse stain of the nuclear contents becomes deeper; small bead-like structures appear within the nucleus. These structures are considerably shorter and are not as clear as in the nuclei treated with hypertonic solutions. However, these experiments likewise seem to support the idea that the resting nucleus consists of chromosomes.

When studying our preparations, we have repeatedly been confronted with the question: What is the distribution of the chromosomes within the nucleus? On the basis of our observations we are inclined to admit that the chromosomes occupy the whole of the nucleus, forming a twisted tangle; they always consist of the chromonema thread with nucleoproteids joined to it. We cannot thus join the assumption advanced by certain writers, namely, that the interphatic nucleus consists only of protein chromonemata free of nucleoproteids.

In view of the fact that our observations have revealed a great similarity of the structure of the nucleus at the stage of earliest prophase, we have counted the number of nuclei showing clear chromosomes; it appeared that they constituted over a half of the total number. This justifies the conclusion that they were no prophatic nuclei.

Our observations thus lead to a conclusion that in the interphatic nucleus the chromosomes are contained in a form of a tangled knot forming the inner frame of the nucleus. If the appearance of the bead-like granules were «paranekrosis», or reversible coagulation, we should then have observed similar aspects at any stages of the nuclear cycle, except for those where the chromosomes are in a state of high spiralization; as a matter of fact, this is not the case.

Under identical treatments the chromosomes were visible with different clarity, and the number of granules was different, too. All this should be taken to mean that the nucleus at interphase undergoes continuous changes, actively participating in the life of the cell.

Received
3. V. 1946.

REFERENCES

- ¹ P. V. Makarov, C. R. Acad. Sci. URSS, **47**, No. 2 (1945). ² C. D. Darlington, Nature, **149**, 66 (1942). ³ T. Painter, Cold Spring Harbor, IV, (1941). ⁴ E. Stedman and E. Stedman, Nature, **152**, 3853 (1943). ⁵ A. B. Румянцев, Культура тканей вне организма и их значение в биологии, 1932. ⁶ Д. П. Насонов и В. Я. Александров, Реакция живого вещества на внешние воздействия, изд. АН СССР, 1940.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS

1946. Volume LIII, № 6

MICROBIOLOGY

ON THE UTILIZATION OF VOLATILE BIOCATALYTIC SUBSTANCES
BY MICROORGANISMS

By M. N. MEISSEL and N. P. TROFIKOVA

(Communicated by A. A. Richler, Member of the Academy, 18. IV. 1946)

N. G. Cholodny has demonstrated quite conclusively by means of simple experiments that higher plants excrete into the surrounding air organic substances favouring the proliferation of microorganisms. He advanced the hypothesis that among other volatile nitrogen- and carbon-containing organic compounds these plants may excrete also various bioactivators, vitamins included (1).

In the present note we adduce the first, so far as we know, experimental proofs of the propagation of a biocatalytic substance by the aerial route.

In studying the functional and morphological significance of vitamin B₁ and of the thiazol and pyrimidin components of its molecule for the microbial cell, our attention was attracted by the circumstance that sterile water solutions of vitamin thiazol (4-methyl-5-β-oxethyl-thiazol), preserved in test-tubes stopped with cotton plugs, lost part of their activity in the course of time. The molecule of this compound is fairly stable, and one could hardly expect it to suffer any chemical alterations. It seemed rather more probable that vitamin thiazol may volatilize.

To verify this possibility use was made of microorganisms needing for their normal development vitamin thiazol, which it is usually necessary to add in the form of water solutions to the synthetic nutrient media when growing them.

As such test-objects we selected *Endomyces Magnatii* which is able to construct the complete molecule of vitamin B₁ from its thiazol part; the mutant of *Neurospora crassa* obtained by D-r Tatum by means of ultra-violet radiation and having lost the ability of synthesizing vitamin thiazol; and finally the classical test-object for vitamin B₁ (or, to be more precise, for vitamin thiazol + pyrimidin)—*Phycomyces nitens*.

The tests were arranged as follows.

Reader's synthetic agar medium was poured out into high Kech dishes, and after its cooling it was seeded all over with the test culture. After this the dishes were placed upside down and several drops of a water solution of vitamin thiazol (usually 1 ng/ml) were introduced under sterile conditions on the inner surface of the lid of the dish. Into the control dishes instead of thiazol we introduced several drops of water.

In a number of tests 250-ml flasks with agar were used instead of Koch dishes. The flasks after their seeding with a microbial suspension were also overturned. In these tests the thiazol solution or the water was introduced into the neck of the flask in small dishes placed on the inner side of the cotton plug.

Finally, with *Endomyces Magnusii* we arranged a test on Reader's liquid medium which was aerated by means of sterile air blown through it. Before entering the medium the air passed through a water solution of thiazol, then through an empty flask, so as to exclude the possibility of drawing into the medium finest droplets of the thiazol solution.

Of course, our experiments involved also a similar control system, in which the air used for the aeration of the medium in control tests was drawn through water instead of the solution of vitamin thiazol.

All the tests gave similar results. The growth of *Endomyces Magnusii* in the Koch dishes with vitamin thiazol produced after 3 days of cultivation a yield of 4.5 g per 10 standard dishes, while the yield of 10 control dishes reached only 1 g. The yield of the same organism in the liquid medium of Reader with aeration by air drawn through a thiazol solution was 400 mg in 200 ml of the medium, and in the control in which the air had been drawn through water, 40 mg, i. e. one tenth of the test yield.

It has been demonstrated in a study published earlier by one of us ⁽²⁾ that *Endomyces Magnusii*, when developing in media to which vitamin thiazol had been added, was able to synthesize great amounts of vitamin B₁ and to accumulate this vitamin within its cells. Such supersynthesis and accumulation of vitamin B₁ is accompanied by a functional and morphological reconstruction of the cells to an anaerobiotic pattern even under aerobic conditions of existence.

The anaerobiotic type is morphologically characterized by a length and size increase of the cell, by an accumulation of glycogen and volutin within it, and by a special structure of its chondriosomic apparatus (its marked hypertrophy). Functionally the anaerobiotic type is distinguished by its increased fermentative ability, by the formation of aceto-ethylic ether and by the accumulation of cocarboxylase and vitamin B₁ in the cells. This functional and morphologic response of *Endomyces Magnusii* to vitamin B₁ (or vitamin thiazol) is, as our investigation has shown, strictly specific and is detectable already at such low concentrations of the substances as $1 \cdot 10^{-7}$.

All these characteristic properties of the anaerobiotic type were found to exist in the cultures of *Endomyces Magnusii* which had been grown over evaporating solutions of vitamin thiazol, while the control cultures consisted of typical aerobiotic cells alone.

The decisive proof of the utilization of vitamin thiazol from the air by the cells is supplied by the results of the quantitative determination of vitamin B₁ performed by us.

While in the control cultures of *Endomyces Magnusii* only traces of vitamin B₁ could be detected, the cultures which had been grown in the presence of evaporating thiazol contained 60-80 micrograms of this vitamin per 1 g of dry weight. Consequently, vitamin thiazol absorbed from the air

is utilized by the organism for building up the entire molecule of vitamin B₁.

Similar results we have obtained with the other two organisms tested, the thiazol-deficient mutant of *Neurospora crassa* and *Phycomyces nitens*, the growth and development of which are powerfully activated by evaporating thiazol, if the other component of vitamin B₁ (2-methyl-5-bromomethyl-6-aminopyrimidin) is present in the medium.

The molecule of vitamin B₁ is, as we know, unstable and easily disintegrates dividing into its thiazol and pyrimidin parts. This process takes place under natural conditions, both when vitamin B₁ is assimilated and decomposed by various organisms and when organic matter containing this vitamin is decomposed. In these cases a part of vitamin thiazol finds its way into the soil in the shape of a water solution, while another part of it may be evaporated into the air and be utilized by other organisms for the biosynthesis of vitamin B₁.

If earlier we knew various volatile compounds which are readily assimilated by microorganisms for their plastic and energetic processes, now we have already grounds to add to their number biocatalytic substances as well.

It seems to us that the strictly specific responses of the microorganisms to vitamins and similar biologically active substances may be applied with success for the detection of infinitesimal concentrations of these substances not only in liquid media but in the air as well. At the same time the volatility of vitamin thiazol discovered by us emphasizes how cautiously laboratory biological tests with substances of this kind should be conducted.

We are at present investigating the possibility of the propagation of entire vitamin B₁ molecules and of the molecules of other vitamins through the air as well as that of their excretion by plants.

Institute of Microbiology.
Academy of Sciences of the USSR.

Received
18. IV. 1946.

REFERENCES

- ¹ N. Choldoy, C. R. Acad. Sci. URSS, **41**, 416 (1946); **43**, 75 and 272 (1944).
- ² M. Meissel, *ibid.*, **41**, 262 (1943).

EXPERIMENTAL MORPHOLOGY

ON THE QUESTION AS TO THE EXISTENCE OF THE «NERVE» LAYER
OF THE ECTODERM

By T. DETTLAFF

(Communicated by I. I. Schmalhausen, Member of the Academy, 21. III. 1946)

I have earlier reported on the results of experiments devoted to the study of the response of the ectodermal layers of *Anura* to the inductive action of the chorda-mesoderm (1). It was found that they respond in different ways, forming respectively the external and internal layers of the neural plate. However, the experimental animals had been fixed at early stages of development, and it was difficult to judge of the further differentiation of their derivatives. On the other hand, we find in the literature on the question contradictory data reported by Schechtman (2), which I intend to discuss in some detail a little later. All this made a further investigation of the problem desirable.

For this purpose the following experiment was arranged. An area of the external layer of the ectoderm was taken from the region of the presumptive neural plate and of the presumptive epidermis of a *Rana esculenta* embryo at the stage of early gastrula and transplanted on to the denudated chorda-mesodermal layer of the neurula of the axolotl, in which the area of its neural plate had preliminarily been eliminated. In another experiment we transplanted on to the denudated chorda-mesodermal layer of the axolotl embryo the corresponding internal layer of the ectoderm of *R. esculenta*. The experiment was consequently a xenoplastic one. Besides the combination *Amblystoma punctatum*—*Rana esculenta*, it was repeated also with the combinations *Triton taeniatus*—*Bufo viridis*, *Amblystoma punctatum*—*Rana arvalis* and *Triton taeniatus*—*R. arvalis*.

The embryos were kept growing as long as possible. The adjoined figures represent one of the most successful instances, in which the embryos survived for a long time. Fig. 1 represents an embryo to which the external layer of the ectoderm was transplanted. Above the chorda we see a thin-walled vesicle with an epithelial cover and a few nerve cells at its inferior surface. It should be noted that in the places where there are nerve cells, the external layer



Fig. 1.

shows no epithelial structure. Its differentiation seems to change in the presence of nerve cells. The same is visible in the experiment involving the substitution of the external layer of *R. esculenta* for the external layer of the neural plate of *R. temporaria* (³); the cells of the external layer penetrate into the thick of the wall of the neural tube and form retort-shaped ependyme-like cells.

Fig. 2 represents a control experiment in which the corresponding internal layer of the ectoderm of an early gastrula had been implanted on the chorda-mesodermal layer of the neurula of an axolotl. A compact mass of nerve tissue was formed from the graft. In spite of the fact that this graft corresponds by its position to the medulla, it does not show the structure typical of the medulla—a large cavity with a thin-walled epithelial cover. But the cavity is absent not in the total length of the graft: in a number of sections we find a small cavity of an irregular contour and atypical shape.



Fig. 2.

Fig. 3 represents a section of the same embryo at a level where part of the nervous system is constituted by the tissue of the host, and another part, by the graft. The difference between the tissues of the axolotl and of the frog is distinctly visible: they differ sharply both in the size of the nuclei and in the intensity of staining. Thus, this experiment shows with great demonstrativeness that the isolated external layer forms under the influence of the inductive action of the chorda-mesoderm a thin-walled tube containing both an epithelial wall and a small quantity of nerve cells.

while the internal layer forms a more or less compact mass of cerebral tissue. As to the few nerve cells forming after the transplantation of the external layers one can think that they might have arisen from a small number of cells of the internal layer transplanted together with the external layer. Of course, one cannot be absolutely certain of this. We had the occasion to observe a similar phenomenon when isolating the external layer of the rudiment of the chorda (⁴); amidst the epithelial tissue individual chordal cells made their appearance in some of the cases. In this case, however, there can be no doubt about the accidental introduction of some cells of the internal layer.

At earlier stages, the transplanted external layer formed a thin or a thicker epithelial wall. In one case the transplanted external layer did not coalesce harmoniously with the host organism and it formed a thin epithelial layer under the epithelium and around the cerebrum, *i. e.* it showed an obvious tendency to epithelize the underlying tissues. The internal layer produced in every host embryo a compact and voluminous area of cerebral tissue. Special interest should be attached to one case in which the transplanted internal layer has not been overgrown by the host's epithelium; its surface acquired the typical structure of epithelium and it covered not only the area of the defect but a larger area overlapping the surface of the host's nervous system. In one of the experiments we transplanted on to the chorda-mesodermal layer the internal layer of the chorda region instead of the internal layer of the ectoderm. In sections from this case we see close to the chorda of the host another chorda developed from the graft—no transformation of the chorda into nervous tissue has occurred.

The results of the experiment described above coincide with those of the experiment with the separation and transplantation of the layers of the neural plate at the stage preceding the formation of the neural groove, when its determination has already taken place but its rolling-up into a tube has not yet begun.

It follows that the result does not depend on whether the determination of the layers of the ectoderm takes place when they are isolated from one another, or when they are joined. Part of these experiments have already been published (⁵). I shall report here on additional experiments involving the separation of layers of the medulla. The technique of these experiments was the same as the one reported in regard to the eye (⁵); the test-object used was *Bufo viridis*; stage, late gastrula—early neurula; site of transplantation, the pharyngeal cavity of the embryo at the stage of early caudal bud or superficially, on to the side of the embryo at the stage of early neurula. When we transplanted the internal layer, there developed a cerebral tissue, compact or often with an atypical cavity arising as a result of regulation (40 tests). In the cases where the graft had been transplanted on to the ventral side sometimes ear vesicles were induced (in 41 embryos).



Fig. 3.

When we transplanted the external layer in a number of cases there arose a vesicle, one part of the wall of which showed an epithelial structure, while the other represented thin-walled cerebrum. The quantity of cerebral tissue in these cases varied from an almost total absence of it to an almost typical cerebrum. The last should be regarded as being due to deficient experimentation. What is important is that in several cases we obtained neuro-epithelial hollow vesicles containing scanty nerve cells. In 8 cases the transplanted external layer remained in the composition of the cover epithelium differing in nothing from it. In 2 cases there lies under it a very small group of nerve cells. Ear vesicles were not induced by transplantation of the external layer. Finally, in 2 of the cases in which transplantation took place on the pharyngeal cavity, a complete separation of the epithelial and nervous parts occurred: an epithelial vesicle, free of any nerve cells lies in the pharyngeal cavity, while a small group of nerve cells form a plot of cerebral tissue in the wall of the pharynx. I must remind the reader of an analogous phenomenon observed after the transplantation of the layers of the chordal rudiment, when the epithelial part remained within the pharyngeal cavity, while the chordal cells lay within the wall of the body (⁵).

It should not be let out of consideration that it must be much more difficult to achieve a complete separation of the layers of the neural plate or of the ectoderm in *Bufo viridis* than in e. g. *Rana esculenta* or *Rana temporaria*, since in the experiments with the rudiment of the chorda, when we isolated the external layer, most of the mixt cases (epithelium + chorda) belonged to those performed on *B. viridis*. We did not succeed in completely separating the layers of the neural plate in it within the limits of the eye rudiment. Thus, the more arduous experiment on an inductive action on isolated ectodermal layers may be substituted by the experiment with the separation of the layers of an already determined neural plate.

Let us return now to the experiment reported by Schechtman⁽²⁾, which in its idea is analogous to mine. Schechtman compared the results of inductive action on ectoderm without its internal layer with those on unimpaired ectoderm. For this purpose he excised a fragment of ectoderm from *Hyla* at the stage of late blastula, dissected it parallel to the surface and eliminated the internal part. The external part with some cells of the internal layer adhering to it (as Schechtman himself emphasizes, the separation was incomplete) he superimposed on the dorsal lip of another gastrula. After their coalescence the area of artificially thinned ectoderm together with the underlying chorda-mesoderm was reimplanted at its former site in the gastrula, and the embryo was allowed to perform the processes of gastrulation and neurulation. In the control test an area of the dorsal lip was made to coalesce with unimpaired ectoderm. In both sets, growth of the nervous system was obtained in an equal percentage of cases and without any delay. However, such a result had to be expected, since the elimination of the cells of the internal layer was incomplete. On the other hand, in the course of the transplantation of the area tested back on to the gastrula the author was unable to exclude the bringing in of the cells of the internal layer from the surrounding ectoderm. Finally, even the external layer alone should show under the inductive action of the chorda-mesoderm the phenomenon of induction, the characteristic elongated cells could be produced and their closing together to form a tube would be achieved. However, this nervous system would be imperfect. The author seems not to have considered this complex character of the formation of the neural plate⁽⁴⁾ in the *Anura*, at least he does not make any note of it.

On account of what has been said above, the results of my experiments seem more conclusive and allow me to speak of the internal and external layers of the ectoderm possessing different form-building potentialities, the terming of the internal layer a nerve layer acquiring an additional justification in these experiments. In spite of that in the *Anura* both layers of ectoderm participate in the formation of the nervous system, the main mass of nerve tissue originates from the internal layer. The external layer takes an active part in the formation of the cavity of the cell layer paving it; it forms the epithelial cover of the cerebrum and the basal plate. There is little probability of the formation of neuroblasts from it.

The A. N. Sewertzov Institute
of Evolutionary Morphology,
Academy of Sciences of the USSR.

Received
21. III. 1946.

REFERENCES

- ¹ T. Dettlaff, C. R. Acad. Sci. URSS, 28, No. 7 (1940). ² I. Schechtman, Proc. of Soc. Exp. Biol. and Med., 38, No. 3 (1938). ³ T. Dettlaff, C. R. Acad. Sci. URSS, 31, No. 2 (1941). ⁴ T. Dettlaff, *ibid.*, 50 (1945). ⁵ T. Dettlaff, *ibid.*, 50 (1945). ⁶ Т. Детлаф, Тр. Ин-та экпер. морфогенеза, VI (1938).

Approved For Release 2002/07/29 : CIA-RDP80-00926A000200010072-1

2

Approved For Release 2002/07/29 : CIA-RDP80-00926A000200010072-1